#### REPORT RESUMES

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REVIEWS OF FILMS, REPORT OF SOME REVIEWING COMMITTEES.

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# REVIEWS OF FILMS

EM 06693

REPORT OF SOME REVIEWING COMMITTEES

Reprinted from

The Mathematics Teacher

December 1963

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS 1201 Sixteenth Street, N.W., Washington, D.C. 20036



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## Reviews of films

A REPORT OF SOME REVIEWING COMMITTEES

Some films prepared for use in secondary school

mathematics classes and in teacher education classes

are listed and reviewed.

During 1962, three committees of high school and college mathematics teachers met on several occasions to review many of the available mathematics films for junior and senior high school. These meetings were financed by the National Council of Teachers of Mathematics. The results of these reviewing sessions are to be found on the following pages.

Originally, the project director, Joseph A. Raab, contacted approximately one hundred film producers and suppliers, and more than two hundred and fifty films were supplied for reviewing. There are, of course, many more mathematics films available from various sources, but those films which were viewed by the committees seem to constitute a good sample.

Each of the three committees was made up of five or six mathematics teachers who were instructed to examine the films with regard to their mathematical content, pedagogical effectiveness, and technical quality. In addition, where possible, particular uses for each film were to be indicated as well as the general level of the film. In general, the committees followed through with these instructions although it was deemed necessary to edit the reviews somewhat in order to achieve some uniformity in style and to condense some reviews.

Thus, each review represents the composite opinion of five or six mathematics teachers. Generally, they have placed more importance on the mathematical content of the film than on its technical

aspects. Such emphasis is probably well justified since a film which is mathematically correct will be useful in the classroom in spite of some technical faults, while no amount of technical skill can make a useful film from one that is mathematically incorrect. Several committee members have commented that the mathematical quality of many of the films is poor even in cases where a mathematics consultant is listed in the production. This is an unfortunate situation which might be remedied by more effective use of these consultants in the actual production of the film. Moreover, there seem to be relatively few films designed to illustrate a particular topic in less than ten minutes, say, and a great many films thirty minutes long, each covering a great many topics. Animation seems to be used less extensively and effectively than one might expect in view of the number of topics in mathematics which involve motion of some sort. Many members felt that short films on the history of mathematics would be of particular value to the mathematics teacher. In short, the consensus of the reviewers seems to be that more mathematics films should be designed to do tasks that the ordinary classroom teacher cannot do effectively, and fewer films designed to "teach" an entire course.

The following mathematics teachers constituted the three committees. Their concerted efforts, constant attention to the job, and open minded attitude are greatly appreciated.

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Joseph A. Raab, Wisconsin State College

Reviews Editor

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Edward Stephany, State University College, Brockport, New York

Frank Viggiani, Rochester Public Schools, Rochester, New York

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### LIST OF FILM DISTRIBUTORS

Association Films, Inc. 347 Madison Avenue New York 17, New York

Cenco Educational Films 1700 Irving Park Road Chicago 13, Illinois

Colburn Film Distributors 1122 Central Avenue Wilmette, Illinois

Coronet Instructional Films 65 East South Water Street Chicago 1, Illinois

Educational Research Council of Greater Cleveland

Rockefeller Building Cleveland 13, Ohio

Encyclopaedia Britannica Films, Inc. 1150 Wilmette Avenue Wilmette, Illinois

Indiana University
Department of Astronomy
Bloomington, Indiana

International Film Bureau 332 South Michigan Avenue Chicago 4, Illinois

McGraw-Hill 330 West 42nd Street New York 36, New York

Modern Learning Aids 3 East 54th Street New York 22, New York

State University of Iowa Bureau of Audio-Visual Instruction Extension Division Iowa City, Iowa

University of Michigan Films 4028 Administration Building Ann Arbor, Michigan

Walt Disney Productions Educational Films Division Burbank, California

### ALPHABETICAL LISTING AND REVIEWS OF FILMS

Addition and Subtraction of Rational Numbers. See Intermediate Algebra Series.

ADDITION FORMULAS AND DEMOIVRE'S THE-OREM. See Trigonometry Series.

\* The following is a key to the symbols used:

sd: sound jh: junior high school
si: silent sh: senior high school
bw: black and white jc: junior college
co: color sc: senior college

el: elementary a: adult

guide: teacher's guide available manual: student manual available ADVANCED ALGEBRA SERIES. 20 films, 1960. sd, bw; sh, jc; tchrs. of sh.\*

Modern Learning Aids, \$3000.†

Historical Introduction to Algebra. 27 min., \$150; jh, sh, jc; tchrs. of jh, sh.

This delightful first film in the advanced algebra series is ably narrated and would be of interest to algebra students at almost any level.

Algebra is discussed as a generalization of arithmetic and is shown to have gone through the rhetorical, syncopated, and symbolic stages of development. Some great books in the story of mathematics are discussed with photos of the Rhind Papyrus, copies of the Diophantos Arithmetic, and Al-Jabr (from which "algebra" is derived) being shown.

An excellent discussion of a cultivation problem brings out questions of manipulation which are to be answered in later films.

Five Fundamental Postulates of Algebra. 30 min., \$150; sh, jc; tchrs. of sh.

Five fundamental postulates are given, namely: commutative and associative laws for addition and multiplication, and the distributive postulate for multiplication over addition. It is emphasized that these postulates are not theorems. An interpretation of the process of division and the implications of dividing by zero are discussed, along with the ideas of least common multiples and least common denominators.

More grouping symbols need to be used in connection with the fundamental postulates. The principles for multiplying by one and for adding zero are called obvious. The discussion of division by zero is not carefully done. The geometric model used to show that the product of two negative numbers is a positive number is very confusing. The film will be fair for review but is poor for teaching these ideas.

Introduction to Factoring. 30 min., \$150; sh, jc; tehrs. of sh.

After a review of the five fundamental postulates for algebra, factoring forms are specifically stated for monomials with common terms, difference of two squares, and quadratic perfect squares. The ideas of factoring algebraic expressions are developed with reference to the factoring forms. A discussion of errors that occur in cancelling is related to factoring.

Although the review of the previous film on postulates is good, the lecturer fails to use these postulates in the discussion on factoring. These factoring techniques involve memorization of certain forms. Stress is placed on the idea that factorization is simply the mastery of certain rules which are to be applied in nine out of ten cases! The third form of factoring is given as  $A^2 \pm 2AB + B^2 = (A + B)^2$  which is incorrect.

† The prices listed are either purchase price or longterm lease. Rental prices may vary depending on source and for this reason are not listed.



Standard Techniques of Factoring. 30 min., \$150; sh, jc; tchrs. of sh.

The following standard forms to be used for factoring are stated: quadratic perfect squares, quadratic nonperfect squares, sum of cubes, and difference of cubes. The lecturer discusses "splitting the middle term" as a method of handling forms involving the quadratic nonperfect square. Certain wrong techniques which are sometimes used in cancelling are identified.

Apparently, these factoring forms are to be memorized. The formula for factoring a general quadratic form is grossly oversimplified. The display of "wrong techniques of cancelling" is not good.

Simplifying Complex Fractions. 30 min., \$150; sh, jc; tchrs. of sh.

Mechanical operations for handling complex fractions are discussed including: (1) in simple cases, invert the denominator and multiply this times the numerator; (2) simplify the numerator then simplify the denominator before using rule (1). Cancelling is mentioned as a factoring technique when both numerator and denominator have been expressed in simplest terms. Certain examples are provided to show the errors that arise when cancelling across addition or subtraction is performed.

The simplifying of complex fractions is generally void of any logical or mathematical reasoning. That "invert and multiply" is equivalent to the "logical" procedure is shown by giving one example. The language used in this entire film is extremely sloppy.

Linear Equations in one Unknown. 30 min., \$150; sh, jc; tchrs. of sh.

The solution of linear equations is introduced by considering the "distance-rate-time" problem in great detail. A small portion of the film is devoted to the transformation of equations to equivalent equations. Extraneous roots of linear equations are touched upon.

Good techniques are used for setting up an equation although the point of view toward equation solving is old-fashioned. Some checking of results is shown but often the result is simply shown to be reasonable. The treatment of extraneous roots is not clear and too much time is spent on mechanical details which really do not help a student understand how to solve linear equations.

Introduction to Simultaneous Equations. 30 min., \$150; sh, jc; tchrs. of sh.

The solution of two simultaneous linear equations is introduced by a verbal problem. Two methods of solution are discussed—comparison and determinants.

The treatment is definitely not introductory in nature as the title suggests. The "method of comparison" is good but the use of determinants is hastily done. The emphasis on mechanical operations is distinctly inappropriate and too much mathematics is passed by as being obvious.

Determinants and Cramer's Rule. 30 min., \$150; sh, jc; tchrs. of sh.

Two simultaneous linear equations are solved using determinants of order two followed by an analysis of three simultaneous linear equations from which Cramer's rule is derived. Determinants of order three are expanded by a method of "bordering" using first and second columns.

The algebraic ideas are grossly oversimplified and there is too much emphasis on mechanical techniques using determinants. The student may be left with the impression that the use of determinants is always easy and the best technique for solving such systems.

Determinants of Any Order. 30 min., \$150; sh, jc; tchrs. of sh.

A general theorem for expanding a determinant by minors of rows or columns is stated and the notion of minors is fully discussed. Careful attention is given to the general applicability of Cramer's rule to determinants of any order.

Although too much stress is placed on techniques and not enough on understanding, the explanation of minors is very good. Such phrases as "throw the zeros in the first column" indicate the nature of the terminology used.

Introduction to Quadratic Equations. 30 min., \$150; sh, jc; tchrs. of sh.

Verbal problems are proposed whose solutions involve quadratic equations. The problems are actually worked out and the emphasis is on the solution of the quadratic equation by completing the square. A derivation of the quadratic formula follows.

The word problems used to give rise to quadratic equations are good. The method of completing the square is reasonably well done but the emphasis is on the quadratic formula which is to be memorized.

Solving Problems with the Quadratic Formula. 30 min., \$150; sh, jc; tchrs. of sh.

The quadratic formula is used to solve a quadratic equation whose roots are irrational. The square root of a number is approximated by the square root algorithm, by the slide rule, and from tables of squares and square roots. Emphasis is placed on location of the decimal point. The use of tables in approximating cube roots concludes the film.

The algorithm for approximating square roots is given purely as a mechanical process without appropriate foundation. Emphasis is on memorization of the quadratic formula with no mention of the principles underlying it. The simplifying of radicals is done well. The film title is misleading since the major portion of the work is devoted to handling square roots.

Complex Numbers and Roots of Equations. 30 min., \$150; sh, jc; tchrs. of sh.

Analysis of the discriminant includes the cases where the discriminant is greater than zero, equal to zero, and less than zero. Definitions of the square root of negative one, a complex number, and an imaginary number are given. The representation of complex numbers in the number plane is described. The film concludes with a discussion of the nature of the roots of systems of quadratic equations.

Although i is defined, very little is done with operations with complex numbers. The complex number plane is handled in a very sketchy manner. The discussion of the nature of the discriminant of a quadratic equation is satisfactorily

handled.

Introduction to Graphs of Equations. 30 min., \$150; sh, jc; tchrs. of sh.

Systems of quadratic equations are discussed along with a system of a quadratic and a linear equation. The use of graphs in interpreting systems of quadratic functions is discussed.

The technique of substituting a linear equation in a quadratic is poorly done. No satisfactory definition of a "quadratic system" is given.

Graphs of Quadratic Equations. 30 min., \$150; sh, jc; tchrs. of sh.

Attention is given to the graphical interpretation of circles, ellipses and parabolas. The discussion is an extension of the presentation in the preceding film. The technique of completing the square is employed in graphing these functions in order to determine a set of transformed axes to reduce the functions to a standard form.

Entirely too much is attempted in this film with the result that the viewer may not understand the basic differences between the graphs of circles, ellipses, and parabolas. The use of the technique of completing the square is satisfactory but is overwhelmed in the bulk of material.

Theory of Equations and Synthetic Division. 30 min., \$150; sh, jc; tchrs. of sh.

Several parabolic functions are graphed using the technique of completing the square. Finding the roots of a cubic equation motivates the introduction of synthetic division which is subsequently used as a test for roots.

Not enough is made of the role of the Factor Theorem and Remainder Theorem. The role of the "missing term" and the linearity of the divisor in synthetic division is not made clear. The viewer is likely to be confused by the film.

Solution of Equations Beyond the Second Degree. 32 min., \$150; sh, jc; tchrs. of sh.

A quartic equation is used to motivate discussion of the Rule of Signs as well as other rules which enable a student to carry out a limited search for rational roots.

Unfortunately, the discussion here is almost completely centered on rules with very little analysis of the relationship between coefficients and roots of the polynomial equations. The result is a film on manipulation and technique.

Permutations and Combinations. 30 min., \$150; sh, jc; tchrs. of sh.

The meaning of permutations and combinations is discussed. The basic formulas for number of permutations of n different objects taken r at a time, without repetitions; permutations of a set of objects not all different; and number of combinations of n different objects taken r at a time are derived and discussed. These formulas are then applied to problems with the admonition that restrictions are often necessary in order to carry through with these formulas.

For a person who is already well acquainted with these topics the film should be acceptable. The derivations of the basic results are not particularly well done. We suggest the film be

used for reviewing only.

Nature of Logarithms. 30 min., \$150; sh, jc; tehrs. of sh.

A complicated multiplication problem is used as motivation for the study of logarithms. Mantissa and characteristics are discussed carefully. The emphasis is on the base ten. The use of positive mantissas with negative characteristics is discussed.

Although the development is consistent with the traditional approach, we feel the stress should be placed more heavily on the underlying

mathematical notions.

Using Logarithms in Problems. 30 min., \$150; sh, jc; tchrs. of sh.

Logarithms with arbitrary bases are discussed including the definition of  $\log_a c = b$  as  $a^b = c$ . Such logarithms are used to solve problems.

The unfortunate inference that the viewer will draw is that the primary purpose of logarithms is to simplify computation. The film will be satisfactory for review purposes.

Computing Logarithms from Arithmetic and Geometric Series. 30 min., \$150; sh, jc; tchrs.

A short history of logarithms is given as well as an explanation of the derivation of the tables. Interpolation is discussed at some length as well as other topics in the use of tables of mantissas.

The historical introduction for the evaluation of logs for tables is good. The tie-in with the previous film, Nature of Logarithms, is timely. The title of the film is misleading since very little discussion centers on these series. The major use of the film should be for review.



Adventures in Number and Space Series. 9 films, 1958. sd, bw; el, jh, sh, a; tchrs. of el, jh; Association Films, Inc., \$1,250.

The series consists of nine films originally produced for television. Although a variety of topics are presented, the series is made contiguous by the presence of Mr. Bill Baird and his marionettes. The resulting vehicle is very effective as a device to arouse interest in mathematics. A note of levity is constantly provided by the discourse, antics, and predicaments of the marionettes. Mr. Baird provides lectures and, at times, problem solving techniques for getting the marionettes out of their scrapes.

The props and devices used in the film will not generally be found in the classroom. Thus, the individual films provide much in the way of motivation and introduction to new topics that would not ordinarily be possible. On the other hand, very little teaching of substantial subject matter will be accomplished directly from use of these films due to the elementary level of treatment. However, the amount of interest generated should easily carry over to regular classroom instruction and give real support to more advanced levels of treatment.

The series has been very well edited on the whole. We found little which could be added or deleted to improve the resultant product. Camera technique is excellent throughout the series and the result is a series which is technically very good.

The most receptive audiences for the series would probably be found at the seventh-through ninth-grade levels. At these levels the films will be good motivational devices for the introduction to algebra, geometry, trigonometry, statistics, and other areas. However, we suggest caution in relying on them as sources of mathematics content as indicated above. The mathematical notions treated are elementary and are not always well developed or connected.

How Man Learned to Count. 30 min., \$150; el, jh, a.

The following notions are dealt with: the elementary reasons and ideas for counting, the Egyptian numeration system up to thousands, the Roman and Babylonian numeration systems, the counting board, the abacus and modern calculators, and the origin of zero.

The reason for counting and having a numeration system is motivated through a classical problem involving cavemen bartering for an ax in terms of "one-two-and a heap" of shells. The difficulties in some notational systems are exemplified by a Roman general attempting to reach a decision about the total number of spears he needs to outfit his army when he knows the number of columns of men and the number of men in each column.

The major portion of the film is devoted to developing the rules governing the counting board. Examples are given and solved by addition using marbles (stones) and grooves in a board (sand). It is mentioned that the counting board was the beginning of the modern decimal system. The abacus is explained and problems are solved on it in competition with a modern electric calculator.

An overwhelming majority of historians believe that the Arabs had little if anything to do with the invention of a zero symbol. Their chief contribution was transmitting the zero symbol to the Western World.

Quicker Than You Think. 30 min., \$150; el, jh, a.

This film is essentially concerned with a development of the binary system, done rather quickly. The motivation for the study of such a system is provided by a chart from which a person's age might be deduced when the person selects the various columns in which his age appears. The presentation is at first a bit awkward, but as the film progresses, there is a good set of devices used to introduce the notion of the binary system. Modern computing machines are demonstrated by visiting a laboratory in which a Westinghouse computer is used, and a brief description of the mechanics of its operation is given.

Mysterious X. 30 min., \$150; el, jh, a.

This film gives good motivation for algebra. Examples of the use of algebra in life are given with the use of formulas exemplified by: The "cricket chirp" formula for temperature, Newton's laws, and Einstein's  $E=mc^2$ .

The presentation defines algebra as a set of laws governing numbers and states some of the laws—for example, the commutative laws. Solution of equations is poorly done since it is being done intuitively rather than by taking advantage of the laws previously considered and named. The word "variable" is not well defined.

This film might be usable at the fifth-through ninth-grade level.

What's the Angle. 30 min., \$150; el, jh, a.

What's the Angle is an historical introduction to the notions of a right triangle and some of the consequences thereof. The film is motivated by the marionettes' confusion over the shape of a baseball diamond, that is, its being a square rather than what they thought a diamond should look like. This leads into the notion of a right triangle and from this Mr. Baird proceeds into a discussion of the ancient pyramids of Egypt. The 3-4-5 relationship is mentioned and the famous rope stretchers are considered in connection with the construction of the pyramids as well as with the beginnings of plane surveying along the Nile. Map and map-making follow with some simple ideas of projective geometry being described through the use of a light source and a translucent sphere.

The film should be usable to aid in introducing geometry although the terminology used may be a bit advanced for a simple introduction.

Some care should be taken by the user to check the understanding of the students' for those words used but not defined in the film.

Arrangements and Combinations. 30 min., \$150; el, jh, a.

This film provides a very interesting and clever introduction to combinations. The general theme of the film is to give careful examples of the uses of combinations in problem solving. Some situations are: the various positions of trains of cars, different clothing changes, the number of menus from a few food items, entry of a present-day newspaper puzzle wherein 1,000,000 solutions would have to be entered to guarantee a winner, and several more.

Greatest use of this film is suggested in Grades 7 through 9. The presentation, though very interesting, is probably too frivolous in nature to provide much in the way of mathematical concepts.

How's Chances. 30 min., \$150; jh, a.

This film will serve as a good introduction to elementary topics in probability and statistics. It refers to the sampling technique of predicting election outcomes. The binomial and normal distributions are discussed and various applications, such as army supplies and a supermarket, are presented. Motivated by a discussion of an incompleted game of points, the marionettes use simple probability. The film provides excellent motivation for elementary study of probability.

Stretching Imagination. 30 min., \$150; jh, a.

Notions of topology are discussed in this film. A fairly intuitive definition of "topologically equivalent" is given and a few examples involving "outside" and "inside" are mentioned. The illustrations include the cup and donut surfaces. Industrial applications are touched upon with items like the roofs of cars and the pressing of plastic materials. A problem involving a paperboy and his shortest delivery route is also mentioned.

The fundamental equation for a polyhedron which expresses the relationship between the number of edges, vertices, and faces is given by an algebraic formula, i.e., e+2=v+f. The mobius strip is discussed, as well as the problem of removing one's vest without removing one's coat. This film might be very useful for mathematics clubs at the high school and junior high school levels. The presentation is not given on quite such an elementary level as many of the others in the series.

Sign Languag:. 30 min., \$150; jh, a.

This is an introductory film on trigonometry. The first discussion involves the tangent relation given as a ratio of rise to run. Applications of this ratio are suggested in navigation and artillery. The use of tables is mentioned for computational purposes. Mr. Baird's enunciation of 101 thousandths comes out "101 thousands."

The sine curve is investigated later, although it is presented as disjoint from the preceding discussion of tangent. Much of the vocabulary in the film is used rather glibly, such as the words period, loudness, length and periodicity. Actually, some improper relationships are suggested in this film which make it usable only as an introductory film for a lower level treatment of trigonometry.

Careers in Mathematics. 30 min., \$150; jh, sh; tchrs. of el, jh, sh.

This film is a short summary of the eight preceding films and points out that there are careers for women as well as for men in mathematics. Essentially nothing new is presented in this last film of the series and we do not recommend its use separately from the series even for the possible benefit of prospective mathematics students. The film provides a too-brief review of previously discussed topics using references to the earlier films in the series.

ALGEBRA AND POWERS OF TEN. See Special Lessons in Physics.

ALGEBRA OF POINTS AND LINES. See Intermediate Algebra Series.

ALGEBRAIC AND COMPLEX FRACTIONS. See Intermediate Algebra Series.

A Plus B Squared. 1954. sd, bw; 10 min.; jh; International Film Bureau, Inc. \$50.

Finding the area of a square motivates the early discussion of  $(a+b)^2$ . During the presentation the narrator says, "We know how to multiply numbers but how do we multiply letters?" Such language is poor, especially for a film. It is geometrically shown that the area of a square of side-measure a+b is  $a^2+2ab+b^2$  but no mention is made of the distributive and commutative laws when  $(a+b)^2$  is expanded algebraically. Most teachers could probably present this material as well as it is presented here. In spots the sound was poor on the film used by the reviewers.

ARITHMETIC: ESTIMATING AND CHECKING ANSWERS. 1962. sd, bw; 11 min., el, jh; Coronet Instructional Films, \$60.

After an introduction showing the need for estimating answers, procedures for rounding off numbers are presented and applied in illustrated word problems using large numbers and using decimals. Checking the four fundamental processes is stressed.

The type of word problems and the method of illustrating them would appeal to sixth-grade students as an introduction to the topic.

ARRANGEMENTS AND COMBINATIONS. See Adventures in Number and Space.



AXIOMS IN ALGEBRA. 1960. sd, co, 13 min., jh, sh; International Film Bureau Inc., \$135.

The axioms discussed in this film are not the axioms of a field but are the traditional version of Euclid's axioms for addition, subtraction, multiplication, division, powers, and roots. The film does not point out that subtraction and division axioms are redundant nor does it restrict the root axiom to principal roots. No recognition of the commutative and associative laws is made. A great deal of effort is made to motivate the learning of each axiom using "practical" illustrations. The narrator speaks of subtracting trucks, adding cages, and multiplying pages which will serve to indicate the lack of consistency with current usage of mathematical language.

Although the photographic techniques are good, this treatment of the axioms in algebra

leaves much to be desired.

BASE AND PLACE. See Understanding Numbers.

BIG NUMBERS. See Understanding Numbers.

CAREERS IN MATHEMATICS. See Adventures in Number and Space.

CHAIN OPERATIONS. See Engineering Computation Skills: The Slide Rule.

COMPLEX NUMBERS AND ROOTS OF EQUATIONS. See Advanced Algebra Series.

COMPOSITE AREAS. See Engineering Computation skills: The Slide Rule.

COMPUTING LOGARITHMS FROM ARITHMETIC AND GEOMETRIC SERIES. See Advanced Algebra Series.

CONCEPT OF A FUNCTION. See McGraw-Hill Teacher Education Series.

CONSTRUCTION OF BASIC SCALES. See Engineering Computation Skills: The Slide Rule.

Conversions. See Engineering Computation Skills: The Slide Rule.

COSECANT, SECANT AND COTANGENT. See Trigonometry Series.

DECIMAL NUMERALS. See Junior High Film Series.

DETERMINANTS AND CRAMER'S RULE. See Advanced Algebra Series.

DETERMINANTS OF ANY ORDER. See Advanced Algebra Series.

DEVELOPING AND SOLVING LINEAR EQUATIONS. See Intermediate Algebra Series. DISCOVERING SOLIDS. 5 films, 1959. sd, co or bw; jh, sh, jc; tchrs. of jh. Cenco Ed. Films, Inc., bw \$375, co \$750.

Solids in the World Around Us. 5 min., bw \$75, co \$150; jh, sh, jc.

This film shows solid geometric form in everyday life through familiar objects such as flowers, butterfly wings, and even the shell of a turtle. In the manner of an art exhibit, the film shows modern geometric figures such as structural steel, manufactured goods, and rockets. Definitions of point, line, radius, a sector, and other terms are given near the conclusion of the film.

The use of natural and artificial art is very effective and will make the film quite useful for motivation in geometry. Although the film contains little explaining, it is definitely a worth-while and enjoyable film.

Volumes of Cubes, Prisms, Cylinders. 5 min., bw \$75, co \$150; jh, sh; tchrs. of jh.

After an examination of solids that exist in the world around us, a cube is defined to be the basic "unit" used in the calculation of volumes. Rectangular prisms are seen to be made up of a series of cubes. Various odd shaped figures are made and their volumes computed by counting the number of cubic units from which they were constructed. An excellent development of the formula for the volume of a rectangular prism follows. The formula for the volume of a cylinder is developed from the relationship already established for the prism.

The development of these formulas is excellent and easily followed. The illustrations and practical applications are well chosen.

Volumes of Pyramids, Cones, and Cylinders. 15 min., bw \$75, co \$150; jh; tchrs. of jh.

Volume is introduced as it is seen in every-day life with animation being used to illustrate that the volume of a prism is equal to area of base times height. The formula for the volume of a pyramid is thoroughly illustrated and leads to the formula for the volume of a cone. Finally, in the same careful manner, the formula for the volume of a sphere is derived.

Animation is used to excellent advantage and the film is definitely a "must" for almost

any group studying volume.

Surface Areas of Solids, Parts I and II. 15 min. each, bw \$75, co \$150; jh, sh; tchrs. of jh.

In the first film, surface areas of cubes, prisms, and pyramids are considered, drawing applications from real life situations. The second film considers surfaces of cylinders, cones, and spheres which are shown to be surfaces of revolution. In both films, the formulas are developed through animation and applied to everyday situations.

While the development of the formulas is well done, those on surfaces of revolution will require additional amplification by the teacher. In any case both films will make excellent introductions to the topics.

Donald in Mathmagic Land. 1959. sd, co, 26 min., el, jh, sh, jc; tchrs. of el, jh, sh; Walt Disney Productions, \$250 (10 yr. lease).

Donald Duck enters a fantasy land of animated numerals and geometric forms and is guided in his journey by a "Spirit of Adventure" whose articulation is much clearer than Donald's. The relation between length and pitch of a vibrating string serves as an excuse for a visit to a Pythagorean jam session. Animated diagrams of the Pythagorean's symbol, the pentagram, show its connection with the golden rectangle and the occurrence of these forms in art and nature. After a discussion of games, including some views of expert billiard shots, the scene shifts to a miscellany of plane and solid geometric forms and their physical applications. Repeating pentagrams within pentagrams and other similar situations are used to introduce the notion of infinity.

The viewer will be attracted by the music, art, animation, and humor rather than by the mathematics. There is probably some motivational value here but an extensive follow-up will be required in order to teach mathematical concepts. Nevertheless, the film is good entertainment for almost any age level.

Double and Half Angle Formulas. See Trigonometry Series.

EARLIEST NUMBERS. See Understanding Numbers.

EFFICIENT OPERATIONS I. See Engineering Computation Skills: The Slide Rule.

EFFICIENT OPERATIONS II. See Engineering Computation Skills: The Slide Rule.

EIGHT FUNDAMENTAL TRIGONOMETRIC IDEN-TITIES. See Trigonometry Series.

ELECTRONIC COMPUTERS AND MATHEMATICS. 1961. sd, co, 25 min.; jh, sh, jc, a; tchrs. of jh, sh; bw \$110, co \$220.

The history of computers is shown in this film from finger counting, use of pebbles, the abacus, to modern electronic giants adaptable to many purposes. The binary system is explained and compared with the decimal system. Many illustrations of working computers are given, identifying the major components such as input, storage, processing, and output units.

Although the film is very interesting and stimulating, the emphasis is on the mechanical and vocational aspects of computers rather than the mathematical aspects. This does not detract from its use as a film for motivation.

ELEMENTS OF TRIGONOMETRY. See Special Lessons in Physics.

ENGINEERING COMPUTATION SKILLS: THE SLIDE RULE. 15 films, 1960. sd, bw; sh, jc, a; Bureau of Audio-Visual Instruction, State University of Iowa, \$1,025; guide, manual.

The series is best suited for use in service courses designed primarily for pre-engineering or science areas where interest is in the manipulative or technical skill in the use of the slide rule. There are no mathematical notions derived or developed with any degree of rigor in any part of the series. We feel that the presentation is well organized, that the lecturer is well prepared, and that his use of props is generally quite adequate.

The mathematics is basically correct. There are some misuses of terminology, and a question concerning the cancellation of units arises in one or two films of the series. The lecture and demonstration methods of instruction are appropriate for the subject matter and seem to be paced so that the average high school or college student should be able to follow along easily. The devices used for instruction are very good and are not generally found in the classroom, especially the chalkboard faced slide rule used in demonstrating the construction of scales. Most of the films end with a brief but thorough review of the topics discussed in that film.

We recommend the series as very useful in teaching skill in the use of the slide rule. The first two or three films could be used separately but the remainder of the series should be used in a consecutive sequence.

Construction of Basic Scales. 23 min., \$75; sh, jc, a.

The construction of the C, D, and A scale is very carefully done. A chalkboard faced slide rule is skillfully used in the film. This film might be used separately from the remainder of the series whenever an explanation of the means of construction of the simple slide rule is desired. Possibilities for use by mathematics clubs or for enrichment programs also exist.

Multiplication and Division. 30 min., \$75; sh, jc, a.

This film shows the two standard demonstration slide rules of the series with the CI, CF, DF, and CIF scales. The construction of these scales and their relation to the C and D scales are briefly noted. The film does a good job of exhibiting the slide rule as a tool and showing its application to problems in mathematics. It can be used separately from the remainder of the series.

Chain Operations. 28 min., \$75; sh, jc, a.

In this film the lecturer uses the entire family of the C and D scales to solve some simple problems in multiplication and division.



We do not recommend the use of this film separately from the previous film because the identification of the scales is made in the earlier film. The procedure and treatment is quite formal, although the devices are excellent. The positioning of the decimal point is done by an approximation procedure rather than a technique involving scientific notation.

Ratio Problems. 29 min., \$75; sh, jc, a.

By solving problems involving ratio, the lecturer gives a good example of the finesse and ingenuity required of the skilled slide rule operator. He insists on the requirement that both the hairline and slide must be moved no more than once in a given operation, hence calling for some ingenuity on the part of the operator. The presentation then proceeds to show a single setting method involving the C and D scales to solve problems involving proportionality.

Conversions. 30 min., \$75; sd, bw; sh, jc, a.

The question of converting from one system of units to another is discussed. The various uses of equality have not been discussed in any of the previous films; however, in this film, the lecturer makes a distinct point of "1 ft. \neq 12 in." The reason given for such-actetoment is that "1 \neq 12" and "ft. \neq in." The need for this particular illustration is not clear and as a convenience, portions of this notion are discarded later. We recommend that the concept and technique of "cancelling units" be thoroughly discussed with the intended viewers before this film is shown.

Squares, Cubes, and Roots. 30 min., \$75; sd, bw; sh, jc, a.

This film contains an adequate presentation of the technique of use and application of the A, B, K, R<sub>1</sub>, and R<sub>2</sub> scales. Simple illustrations are used to find the second and third powers and second and third roots of numbers.

Efficient Operations I. 27 min., \$75; sd, bw; sh, jc, a.

The theme of this film is the use of shortcuts in using the slide rule. The film opens with a discussion of the importance of the study of logarithms. In this discussion the lecturer refers to a set of examples on the chalkboard which are too numerous for easy comprehension and seem contrived. We feel that this discussion should have been placed much earlier in the series since the notion of logarithm is at the foundation of the theory behind the slide rule.

Manipulations of the slide rule to solve equations in the form  $y = kx^2$  are demonstrated for slide rules with A and B or  $R_1$  and  $R_2$  scales. The manipulation necessary for finding the area of a circle in one setting is shown, as well as a method of finding the circumference of a circle.

Efficient Operations II. 32 min., \$75; sd, bw; sh, jc, a.

The content of this film is similar to the previous one, and the theme, namely that of short cuts in the use of the slide rule, continues. The topics considered range from area comparisons to approximate solutions of a quadratic equation. The latter topic is restricted to those quadratic equations where one root is known or, at least, can be estimated. In the summary, the comment is made that the intent was to obtain solutions to problems not necessarily directly adapted to the slide rule.

Raising Numbers to Powers. 31 min., \$75; sd, bw; sh, jc, a.

The film deals with the construction and use of log log scales discussed to an extended degree. When factoring an expression such as  $8^5$  to obtain  $8^3 \cdot 8^2$ , the lecturer calls this "factoring the exponent" which seems to us to be inappropriate. The same phrase is used to describe the sentences:  $8^5 = (8^{2.5})^2$  and  $8^5 = 2^5 \cdot 4^5$ . In the summary at the conclusion of the film, the lecturer uses what we feel is more appropriate terminology in dealing with this issue.

Roots and Exponential Equations. 29 min., \$75; sd, bw; sh, jc, a.

The film begins with a good review of the previous film and then proceeds with the general problem of extracting roots by means of the slide rule. The symbol  $N^{1/r}$  is written on the chalkboard on several occasions and the lecturer leaves us with the impression that r is the root of N rather than the index of the root. We are also left with the feeling that the extraction of roots is simply a mechanical process since no mention is made of the theoretical background involved. The relation between the log log scales and the natural logarithm of a number is not made clear in this film although an example is given involving this relationship.

Trigonometric Scales. 31 min., \$75; sd, bw; sh, jc, a.

An introduction to the use of the slide rule in trigonometry is given in this film. The trigonometric scales of the slide rule are constructed with care and in a manner which is descriptive of the nature of the sine and cosine functions. Graphs of the sine function and log sine function for small arguments are shown and are quite adequate for the purpose of the film.

Decimal notation for degrees and fractions thereof are justified on the basis that they are more useful in engineering and science, and the scales on the demonstration slide rule are labeled accordingly. The cofunction relationship between sine and cosine is described in detail and is used to justify the use of the S scale to find values of the cosine function. The T and ST scales are constructed and discussed.

Little computation is performed in this film and consequently it moves rather rapidly. This

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film might be used as an introduction to the use of the slide rule in trigonometry by giving an explanation of the construction of the trigonometric scales.

Right Triangles. 32 min., \$75; sd, bw; sh, jc, a

Methods of solving right triangles are discussed with emphasis on the problem: given two sides, find the hypotenuse and angles. The sinetangent method, which solves the problem in two settings, and an approximation method, are

derived and demonstrated.

The following quotation, taken directly from the sound track, serves to illustrate the degree of detail used in teaching the techniques of slide rule manipulation as well as to illustrate other features of the series. "... and thirdly we have the relationship that says the angles of a right triangle are complementary. To handle these problems, we often use our sine-tangent technique that is particularly useful when we know the values of the two sides of a triangle and are looking for the hypotenuse. This method we know to be good for all cases of angle A greater than 5.75°. If the angle is less than 5.75°, we must use our approximation method. In the approximation method itself, which is: the hypotenuse is equal to the long side plus the short side squared over twice the long side, is good up to 23°. So we see the sine-tangent method will work down to 5.75° and the approximation method up to 23°. These methods, then, will help us solve for right triangles using our slide rule."

Right Triangle Applications. 31 min., \$75; sd, bw; sh, jc, a.

The film begins with a synopsis of the previous film and the precise steps for calculation in the sine-tangent method are listed. A few very simple problems are worked out using this method from definitions of the cosine function as equal to the "adj" over the "hypot" and sine function as equal to the "opp" over the "hypot."

Mention is made of types of problems involved in practical applications but no example from any of the types is worked out in detail. There are no significant examples of any sort

completed in this film.

Scalars and vectors are used as different concepts but no clear distinction is made between them and no definition of vector notation is given.

Oblique Triangles. 30 min., \$75; sd, bw; sh, jc, a.

A chart is used to coordinate the relationships for the general triangle. The sum of the angles of a triangle is given as 180°; the Sine and Cosine Laws are stated. Fairly trivial geometric illustrations of each of these are presented using the slide rule whenever possible for calculations. We note the absence of any reference to the Law of Tangents. Some of the problems

dealt with could be more easily solved with the slide rule by using the Law of Tangents.

There is an unusual amount of very distracting busywork on the chalkboard at the end of the film.

Composite Areas. 26 min., \$75; sd, bw; sh, jc, a.

The computation of areas of polygons and curves by considering them as composite areas is discussed. The composite area is determined by summing the areas of triangles, rectangles, segments, and sectors. The slide rule is used whenever possible to calculate these partial areas. The constructions and diagrams used for dismemberment are very well executed. No summary of the series is included at the end of this final film.

Engineering Problems. 6 films, 1958. sd, bw; sh, jc; Bureau of Audio-Visual Instruction, State Univ. of Iowa, \$400.

The use of the series is recommended only in situations where qualified staff is not available to teach logarithms or to provide a service course for the techniques of logarithmic manipulation. The films could serve only in a situation wherein the student has no demand for mathematical

theory

Nothing is done in this series which could not be done better in a classroom by most regular faculty members. The lecture method is used throughout the entire series with the result that the presentation is very formal and, in turn, very uninteresting. In the early films the lecturer often mumbles while performing computations at the chalkboard. The lecturer habitually speaks directly to the chalkboard with no microphone provided for such action making it difficult to hear what is said. Generally, the use of the chalkboard is good, and the solutions of examples are presented in step-by-step developments rather than referring to pre-obtained results. The camera techniques are also to be commended although some drawings and sketches are smaller than desirable and make for insurmountable problems for the cameraman. This is especially true of the film on interpolation of mantissas.

The mathematical concepts presented are basically correct, although not generally couched in contemporary terminology. Exceptions to this are the derivations and definitions of the trigonometric functions which are done very well in the manner of circular functions. No applications of logarithms to any mathematics or science are presented. There are many instances of poor phrasing and symbolism. In this category we include the mix up between number and numeral, the omission of identifying symbols such as degree and minute marks and decimals before mantissas, the consistant substitution of the word "power" for "exponent," and the phrasing which leads the viewer to believe in the exactness of decimal approximations to irra-

tional numbers, e.g.,  $\log_{10}e = .4343$ .

A student may achieve some understanding of techniques of logarithmic manipulation and their use in problem solving through the use of these films. However, we suggest that there will be serious confusion about certain mathematical concepts for those students who may be intertested in pursuing careers in mathematics rather than careers in engineering or technical fields.

Logarithms—Characteristics. 26 min., \$75; sh, jc, a.

We cite the following errors: the logarithm of one to the base two is denoted as two; the first power of ten is said to be equal to one. These errors indicate a need for editing. We also call attention to the need for consistency in "round-

ing off numbers."

In this film there is a mix up between "number" and "numeral" which is further complicated by the use of such phrases as "behind the number" and "the plus part of a number." Such statements as "The power of a number is a number," and "In scientific notation, the power of ten is the characteristic of the number," exemplify that "exponent" and "power" are often interchanged.

The illustrative examples seem contrived; they consist of the ordered sequence of the digits 1, 2, 3, 4, 5, 6, and 7 with the decimal point being placed in various positions, e.g., 123.4567, 1234.567, 12.34567, etc. A more random sequence of digits would have been

more effective and illustrative.

Mantissas. 26 min., \$75; sd, bw; sh, jc, a.

The film deals with obtaining the mantissas of logarithms. It gives a good graphic portrayal of linear interpolation to find an approximation to a desired mantissa. The non-linearity of the logarithm function is pointed out as is the partial correction of interpolation through the use of larger tables. We feel it is poor pedagogy to omit symbols for degrees and minutes and decimal points before mantissas.

Logarithmic Operations I, II. 27 min., each, \$75; sh, jc, a.

In these films, the algebraic theorems involving exponents are applied to the definition of logarithm to arrive at the theorems on logarithmic operations. The illustrative examples, however, seem to be contrived and have little meaning to the viewer. The lecturer seems to be mumbling to the chalkboard a good deal to the extent that he appears to be talking to himself.

These films follow the mood of the series by teaching the manipulation of logarithms rather than theory of logarithms. The objective, therefore, seems to be to develop the tool of log-

arithmic manipulation.

In the second film of this pair, log logs are investigated as well as fractional powers of fractional numbers. The definitions of the trigonometric functions are given by means of coordinate geometry. A table of algebraic signs is

constructed for the four most important trigonometric functions and is to be memorized. This again emphasizes the stress being placed in this series on manipulative rules and memorized devices.

Trigonometric Applications. 32 min., \$75; sh, jc, a.

Contained in this film are a brief review of the unit circle definition of the trigonometric functions, statements of the Law of Sines and Law of Cosines without proof of either, and examples of the application of each of these laws. Finally, definitions of sin, cos, and tan are given in terms of a right triangle. The lecture method is continued and the presentation is extremely formal.

Logarithmic Systems. 21 min., \$75; sh, jc, a.

The last of this series compares logarithms of base ten with logarithms of base e. Base e is tied to base ten in such a specific manner that the viewer may be left with the impression that logarithms of base e exist only as related to logarithms of base ten. The derivation of characteristics and mantissas for base e could have been given just as formally as those for base ten and without the use of logarithms of base ten. Unfortunately, the viewer may also have the impression that the logarithm of e to the base ten is exactly 0.4343.

EQUATIONS AND GRAPHS OF THE PARABOLA. See Intermediate Algebra Series.

EQUATIONS WITH UNKNOWNS IN THE EXPONENTS. See Intermediate Algebra Series.

FIVE FUNDAMENTAL POSTULATES OF ALGEBRA. See Advanced Algebra Series.

FORMULAS IN MATHEMATICS. 1960. sd, co, 10 min.; jh; International Film Bureau, Inc., \$110.

A treatment is given of the distance formula D=rt with the example of an airplane in flight. The use of this formula is illustrated.

A perplexing use of language is noticed in this film. For example, the narrator states that ninety miles divided by one hour is ninety miles per hour. This will likely make it seem as if division is an operation on objects other than numbers. The viewer is cautioned to use the same units of measure but is not told why. We find it difficult to conceive of a place in a mathematics program where this film might be profitably used.

FRACTIONS. See Understanding Numbers.

FUNDAMENTAL OPERATIONS. See Understanding Numbers.

GENERAL METHODS FOR SOLVING QUADRATIC EQUATIONS. See Intermediate Algebra Series.

GRAPHING LINEAR EQUATIONS. 1961, sd, co, bw, 12 min.; jh, sh; Coronet Instructional Films, bw \$60, co \$110, guide.

The step-by-step construction of linear graphs is shown in detail as well as the relations between the line graph and the points in rectangular coordinates. Slopes and intercepts are discussed as well as the effect that changes in these parameters have on the graph.

In general the film is good and would be useful for senior high school algebra students but the discussion of slope and intercept will need

extension.

GRAPHS OF PERIODIC FUNCTIONS. See Trigonometry Series.

Graphs of Quadratic Equations. See Advanced Algebra Series.

HISTORICAL INTRODUCTION TO ALGEBRA. See Advanced Algebra Series.

How Man Learned to Count. See Adventures in Number and Space.

How's Chances. See Adventures in Number and Space.

HYPERBOLA, ELLIPSE AND CIRCLE. See Intermediate Algebra Series.

THE IDEA OF NUMBERS: AN INTRODUCTION TO NUMBER SYSTEMS. 1960. sd, co, 14 min.; el, jh; tchrs. of el, jh; International Film Bureau, Inc., \$135.

The development of the number concept is given historically, but confusion arises in this film as to the difference between number and numeral. Systems of numeration such as the Babylonian, Mayan, Arabic, and Roman are discussed as well as calculating aids that have been used. Place value systems are noted with some emphasis on base two. The notion that a number is an idea, not a mark on paper, is not made clear and is further complicated by terms like "three place number." However, with careful correction of these errors by the teacher using this film, we recommend its use because of its technical qualities and its historical material.

IMAGINARY AND COMPLEX NUMBER. See Intermediate Algebra Series.

INFINITE SERIES AND THE BINOMIAL EXPAN-SION. See Intermediate Algebra Series.

THE INTEGERS. See Junior High Film Series.

INTERMEDIATE ALGEBRA SERIES. 24 films, 1959. sd, bw; Modern Learning Aids, \$3,600; sh, jc; sh.

This series of twenty-four films presents a selection of topics usually included in a second course in high school algebra.

Natural Numbers, Integers and Rational Numbers. 30 min., \$150; jh, sh, jc; tchrs. of jh, sh.

The instructor considers the natural numbers and some of their properties. These include comparison of magnitudes, addition, and multiplication along with the allied notions of closure, identity elements, and the commutative, associative and distributive laws. Solution of simple equations is discussed, but the viewer may be left with the impression that -a is a negative number. Reciprocals are used to introduce the rational numbers.

Despite a few misleading examples, the film is usable for algebra classes or for in-service teachers.

Addition and Subtraction of Rational Numbers. 30 min., \$150; sh, jc; tchrs. of sh.

After a brief review of the preceding film, the instructor proceeds to a consideration of the number line. It is indicated that every raticnal number corresponds to a point on the line and the absolute value of a number is defined to be the distance between a point corresponding to the number and the point corresponding to zero.

Although some proofs of theorems involving additive inverses are given, no mention is made of the dependence upon the uniqueness of the additive inverse. The "rules" for addition in terms of absolute value are poorly presented. The convention in connection with the equality of 3-2 and 3+(-2) is not explained.

The content of this film is not as well presented as in the first film, nor is it presented in the spirit of contemporary mathematics.

Multiplication of Rational Numbers. 30 min., \$150; sh, jc; tchrs. of sh.

Theorems concerning multiplication of rational numbers are given, but the basic theorem on the uniqueness of the additive inverse is again slighted. The viewer may also be led to believe that (-a)(-b)=ab is a theorem concerning the product of two negative numbers since the symbol -a is not clearly defined. Division is covered hastily and includes a discussion of the three signs of a fraction, which is not the best way of handling this. Algebraic expressions are considered and the division of polynomials is discussed in a strictly traditional fashion.

Developing and Solving Linear Equations. 30 min., \$150; sh, jc; tchrs. of sh.

The instructor discusses the solving of linear equations with restrictions placed on the variable x. Graphs are used to picture the set of points which satisfy these restrictions. The instructor is not careful in his use of the language when he states "we want x alone" and "'x' is in the parentheses and clearly we must get it out." The checking of a solution is emphasized.

The film could be used to introduce linear equations or as a quick review but should not be used in contemporary mathematics programs.

Solving Simultaneous Linear Equations. 30 min., \$150; sh, jc; tchrs. of sh.

After a brief review of the solving of linear equations in the variable x, conditions involving two variables are discussed by means of examples. The domains of the variables are not specified. The procedure for picturing an ordered

pair of numbers is outlined.

Although a linear function is graphed, no mention is made of the terms "intercept" or "slope." Graphing is illustrated as an approximate method of finding a simultaneous solution of two linear conditions by inspecting the pictures of the graphs. A pair of simultaneous equations is solved algebraically. Nothing is said of the concept of equivalent equations. The solving of simultaneous conditions involving inequalities is not considered.

More Solutions of Linear Equations. 30 min., \$150; sh, jc; tchrs. of sh.

This film deals with the problem of solving verbal problems. Four such problems are worked as examples. These involve a problem on digits, mixtures, and uniform velocities. It is clearly indicated that the solution to the equation is a number while the solution to the problem may be a number of units. The viewer is assured that there is no set procedure in the solution of verbal problems.

This film is very good and could make a valuable contribution to any course in intermediate algebra.

Special Products and Factoring. 30 min., \$150; sh, jc; tchrs. of sh.

Although the instructor uses the word "factor" in explaining the process of "factoring," the distributive law is used as related to factoring a common factor and to the multiplication of binomials. Little is said of the basic use of factoring, that is, to write equivalent expressions in a different form, and too much is said about the time worn gimmicks for factoring the

usual types of quadratics.

In discussing  $a^2-b^3$ , the instructor relies too heavily upon the experience of the student to suggest the factor a-b. More needs to be said about the reasons why  $a^2+b^2$  is not factorable. The instructor evidently assumes that a, b, c, and d are integers only when finding, by trial and error, factors of the form (ax+b) and (cx+d). The domain of these variables should be extended to the rational numbers, real numbers, and possibly even the complex numbers for intermediate algebra students. Another error is indicated by the statement that  $\sqrt{a^2} = a$ , which should be restricted to non-negative numbers.

This film certainly needs much supplementation if it is to be used at any level. Quadratic Equations. 30 min., \$150; sh, jc; tchrs. of sh.

The discussion of  $ax^2+bx+c=0$  as a quadratic equation does not restrict a to non-zero numbers. Use is made of the statement, "If  $a \cdot b = 0$ , then a = 0 or b = 0," but should be clearly stated as a bi-conditional statement. The method of completing the square is handled well by using the statement, " $x^2 = y$  if and only if  $x = \pm \sqrt{y}$ ."

However, the method of completing the square is not shown to be a method of factoring over the real numbers and inequalities are not developed parallel to this work with equalities. More work will be needed with the method of completing the square in order to make the derivation of the quadratic formula meaningful

to most students.

The film could be used to summarize the factoring and completing the square methods of solving quadratic equations if the curriculum design is along traditional lines.

Algebraic and Complex Fractions. 30 min., \$150; sh, jc; tchrs. of sh.

Without proof the instructor states that the reciprocal of a product is the product of the reciprocals and assumes the uniqueness of the multiplicative inverse. No complete proof of

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

 $b\neq 0$ ,  $d\neq 0$ , is given and the instructor "obtains the value" of

 $\frac{a}{b}$ 

instead of obtaining an equivalent expression. Use is made of the commutative and distributive laws but no mention is made of them when they are used. Cancelling is described in terms of dividing the numerator and denominator by the same number without mentioning restrictions. The summary is a restatement of traditional rules of manipulation which serves to indicate that the function of the film will be as a lesson in symbol manipulation.

Because the basic axioms developed in the first film of the series are not emphasized here, the film may be of questionable value to algebra teachers who are stressing the structure of

algebra.

Solving Equations in Fractional Form. 30 min., \$150; sh, jc; tchrs. of sh.

A fractional equation is defined as one in which the unknown appears in a denominator. It is noted that it immediately follows that  $x \neq 0$  since division by zero is not defined. Examples are used to illustrate technique which is set down by three rules:

1. Multiply by the least common denominator.

2. Solve the resulting equation.

3. Check the solutions.

The use of such a list of steps is questionable since it leaves no room for originality and, in fact, the instructor does not always conform to these steps himself.

In general, the instructor does a good job of

handling traditional material.

Algebra of Points and Lines. 30 min., \$150; sh, jc; tchrs. of sh.

The instructor uses the background of the development in the fifth film of this series, "Solving Simultaneous Linear Equations," and begins by asking what the equations of parallel lines have in common. Slope and y-intercept are then discussed. Slope is defined in terms of increments and not defined for lines parallel to the y-axis. It is not made clear whether the term y-intercept refers to the number b or the point (0,b). The equation of a line through two points is discussed in the conclusion of the film.

The instructor does a good job of presenting this material but the film covers too much ground to be used for any purpose other than

review.

Variation: A Lesson in Reading. 30 min., \$150; sh, jc; tchrs. of sh.

The film emphasizes the many verbal expressions we have for the functions defined by equations of the form y = kx and  $y = \frac{k}{2}$ . Of particular interest is a definition of inverse variation in terms of direct variation, namely: y varies inversely as x if and only if y varies directly as the reciprocal of x.

Examples from geometry and physics are used to illustrate variation problems. Quadratic and cubic variation are discussed as well as

linear variation and joint variation.

This film could be used not only in algebra and physical science classes but also in teacher training classes as an example of sound mathematics teaching.

Radicals and the Real Number System. 30 min., \$150; sh, jc; tchrs. of sh.

After a brief review of earlier comments on irrational numbers, a short historical sketch of the discovery of non-rational numbers, and a sketch of  $\sqrt{2}$  as the diagonal of a square of unit side, the instructor extends the numeration system by defining a number for which he has a need and which is not in the present system. This was well done and included a brief comment on imaginary numbers.

Addition of radicals is presented and associated with the distributive law and the rules for multiplying and dividing square roots. The instructor emphasizes an often overlooked notion that  $\sqrt{3}$  has only a decimal approximation

and no decimal equivalent.

Many additions will need to be made by the teacher who uses this film. The fact that  $\sqrt{x^2} = -x$ , if x < 0, should be stressed, as well as the fact that the operations with real numbers are the means to write expressions in other equivalent forms.

Use of this film is suggested for viewing as a summary of material which has already been

discussed in class.

Roots of Higher Order. 30 min., \$150; sh, jc; tchrs. of sh.

A recursive definition of radicals is given and product and quotient of radicals is extended to orders higher than two. Imaginary numbers are introduced as well as complex numbers and their notation.

The film is quite carefully done and could be used by students as a summary for material

covered in class.

Imaginary and Complex Numbers. 30 min., \$150; sh, jc; tchrs. of sh.

Complex numbers are defined as numbers of the form a+bi where a and b are real numbers and  $i^2=-1$ . Addition and multiplication of

All the mechanical aspects of complex numbers are included in this film including a brief discussion of complex numbers plotted on a plane but the really important ideas are missing. Nothing is said about the ultimate objective which is to construct a new system of numbers which will contain the real numbers as a subset, which will have all the properties of the real numbers, and which will contain solutions for equations which have no solution in the real numbers. The construction of the system of complex numbers as ordered pairs of real numbers is not hinted at here.

It is hard to see how this film will be of any help in clarifying the introduction to complex

numbers.

Working with Positive and Negative Exponents. 30 min., \$150; sh, jc; tchrs. of sh.

Integral exponents are defined together with the rules of operation for positive integral exponents. The discussion brings out the rationale behind the definition of negative and zero exponents. A discussion of scientific notation brings out the subject of significant digits.

The film could be used as an introduction to

or review of integral exponents.

Using Fractional and Rational Exponents. 28 min., \$150; sh, jc; tchrs. of sh.

Definitions of fractional exponents are given and shown to have properties consistent with the properties of integral exponents. Some further discussion of the restrictions on the base will be called for from the teacher. Examples show the use of fractional exponents in treating roots of various orders.



General Methods for Solving Quadratic Equations. 30 min., \$150; sh, jc; tchrs. of sh.

A quadratic equation in the variable x is defined as a condition which can be put in the form  $ax^2+bx+c=0$  where  $a\neq 0$  and a, b, and c are real numbers. It is pointed out that the parameters could be complex numbers but that the discussion here will be limited to reals.

The methods of factoring and completing the square are reviewed and the quadratic formula is derived. Relationships between the parameters and the sum and product of roots are given, and the use of the discriminent to determine the character of the roots is outlined.

In general this film is well worth showing to intermediate algebra students.

Equations and Graphs of the Parabola. 30 min., \$150; sh, jc; tchrs of sh.

Graphs of equations of the form  $y=ax^2+bx+c$  and close relatives are considered in this film. The restriction  $a\neq 0$  is not mentioned. Pictures of the various graphs are compared to indicate the effects of changes in parameters. Axis of symmetry is discussed in relation to the process of completing the square, and the term vertex is defined.

The film's best use will be in reviewing the parabola.

Hyperbola, Ellipse and Circle. 30 min., \$150; sh, jc; tchrs. of sh.

A brief review of the parabola is presented but the necessary restriction  $a \neq 0$  is omitted. The point is made that one draws a picture of a graph and does not draw the graph itself. Two general types of hyperbolas are indicated, namely: xy=c, and  $ax^2+by^2=c$  (ab<0). Discontinuity is not mentioned and asymptotes need fuller explanation. Ellipses are discussed and the circle is seen as a special case of the ellipse. Solutions of simultaneous linear and quadratic equations are given by graphing and algebraic techniques. Finally, solutions of two simultaneous quadratic equations are considered and the methods of solution are outlined.

Although this film gives a good presentation of these topics, the competent teacher could do as well in the regular classroom.

Progressions, Sequences and Series. 30 min., \$150; sh, jc; tchrs. of sh.

Arithmetic and geometric progressions are discussed including the four elements of an arithmetic progression, namely: first term, common difference, number of terms, and last term; similarly, the elements of a geometric progression are discussed. A series is defined as the sum of the terms in a progression. Analytic methods for determining the sum of terms of arithmetic and geometric progressions are discussed fully.

The words "progression" and "series" are confused on occasion. Grouping symbols are badly needed in the formula for the sum of a

geometric progression. The development is much too rapid for an introduction to these ideas. Not enough examples are given for either progressions or series.

Infinite Series and the Binomial Expansion. 29 min., \$150; sh, jc; tchrs. of sh.

Using a geometric series in which the number of terms added increases without bound, it is demonstrated that if the common ratio of a geometric series falls in the range from -1 to 1 and if the number of terms increases without bound, then the sum,  $S_{\infty} = a/(1-r)$ . A discussion of limits and repeating decimals follows. The film concludes with a statement concerning the binomial expansion and the application of the binomial expansion to an example.

The instructor fails to define terms used in the discussion. Series has never been carefully defined here. The development of the rationale behind the coefficients in the binomial expansion is entirely omitted. We fear that the viewer may gain the impression that getting a rule or formula is the heart of mathematics. The nature of limits is poorly handled and no mention is made of r=0 in the sum of an infinite geometric series.

Equations with Unknowns in the Exponents. 30 min., \$150; sh, jc; tehrs of sh.

A general exponential equation is given, namely:  $b^x = a$ . It is mentioned that this equation has a solution if b is positive and not equal to one and if a is positive. It is shown that  $b^x = a$  is equivalent to  $\log_b a = x$ . Basic ideas of logarithms are then discussed along with the use of scientific notation. Definitions of mantissa and characteristics are given.

In the equation  $b^x = a$ , no mention is made as to why such an x exists and none as to why  $b^x = b^y$  implies x = y. The property that  $\log(ac) = \log a + \log c$  is given without any explanation. The treatment of scientific notation is poorly done.

Using Logarithms to Solve Equations. 30 min., \$150; sh, jc; tchrs. of sh.

Beginning with  $5^x=3$ , the fact that  $x=\log 3/\log 5$  is derived. This indicates a need for tables of approximations. These tables could be used to find approximate values for products, quotients, and roots of numbers.

The lecturer gives us the impression that each rational number can be written with a finite number of decimal places. "Distance" between numbers is mentioned, and the distinction between ratio and proportion is not made clear. It is pointed out that solutions with logarithms are approximations, and the emphasis on checking the reasonableness of solutions is good.

Interpolation in Trigonometric Tables. See Trigonometry Series.

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Introduction to Factoring. See Advanced Algebra Series.

Introduction to Graphs of Equations. See Advanced Algebra Series.

Introduction to Logarithms. See Trigonometry Series.

Introduction to Quadratic Equations. See Advanced Algebra Series.

Introduction to Simultaneous Equations. See Advanced Algebra Series.

IRRATIONAL NUMBERS. See McGraw-Hill Teacher Education Series.

JUNIOR HIGH FILM SERIES. 6 films, \*\* 62. sd, bw; jh, tchrs. of jh; Educational Research Council of Greater Cleveland, \$720; guide.

Numeration Systems. 30 min., \$720 the set; jh; tchrs. of el, jh; guide.

The film begins with remarks on the positional nature of our numeration system based on ten. Supposing that seven had been originally chosen as a base instead of ten, the lecturer does some counting in base seven and goes on to discuss the role of seven and its powers, addition and multiplication in base seven, and carries out some manipulations. Binary numeration is discussed, including tables of addition and multiplication. The film closes with some implications resulting from choice of greater and lesser bases.

The mathematics in this film is without fault, and the presentation has been done beautifully in a concise and informative manner. The film will find use in junior high school classes and would be valuable for teacher training and inservice work.

The Whole Numbers. 30 min., \$720 the set; jh, sh; tchrs. of el, jh, sh; guide.

The film deals mainly with the whole numbers and emphasizes the difference between numbers and the symbols which represent them. Commutative, associative, and distributive principles, as well as properties of 0 and 1 are discussed. The instructor illustrates the fact that by applying these principles we can cut down on the number of facts that need to be learned.

Elementary teachers might profit from the discussion of computation which shows how the steps involved in the process can be justified. The film concludes with some comments on the association of arithmetic and geometry through the number line.

The film could be used as an introduction to structure for junior and senior high school students and as a teacher training film.

The Integers. 30 min., \$720 the set; jh, sh; tchrs. of jh. sh; guide.

Basically, the film demonstrates a fairly rigorous introduction to negative integers. The need for signed numbers is suggested by the notion of a thermometer with degrees above and below zero. The existence of negative numbers is postulated and it is assumed that the unique solution of a+b=0 is b=(-a). After a brief digression to define absolute value, operations with the integers are illustrated. Proofs are based on fundamental postulates.

The pace of this film makes it more suitable for teacher training than for viewing by students.

The Rational Numbers. 30 min., \$720 the set; jh; tchrs. of el, jh, sh; guide.

It is seen that measurement necessitates numbers in addition to the set of integers. The number called "a divided by b" is defined as the number such that  $(a \div b) \cdot b = a$  for all integers a and b except b = 0. This set of numbers is then named the set of rational numbers and it is seen that the integers form a subset of the rational numbers. Using the basic laws of integers, the properties of rational numbers are derived.

Although the development does not follow a completely rigorous design, the subject is treated quite well and may, in fact, give many teachers a clearer picture of the rational number system. The students will probably find the film most satisfactory as a review.

st satisfactory as a review.

Decimal Numerals. 30 min., \$720 the set; jh, sh; tchrs. of el, jh, sh; guide.

The positional numeration, base ten, is extended to include not only numerals for whole numbers but also all rational numbers. This is accomplished through the process of division. Repeating and non-repeating decimals are noted making the point that if is a rational number, then has a decimal numeral which either terminates or is periodic. The converse of the preceding conditional is also indicated as being true. The teacher's film guide will be of great help in using this film.

We heartily recommend the film for viewing by all teachers of elementary and secondary mathematics. Although it proceeds at a rather rapid rate, it would be suitable for pupil viewing where the student has been well prepared and especially suited for review. The film provided the reviewers had poor sound and synchronization in spots.

Language of Algebra. 30 min., \$720 the set; jh, sh; tchrs. of jh, sh; guide.

Teachers who feel that the modern approach to algebra is not for them should view this film. The lecturer gives a concise introduction to algebra which is very illuminating. Open sentences, set selector, and solution set are discussed as well as variables and domain.

The film will be excellent not only for teachers, but also as an introduction to algebra for beginning students.

ERIC

LANGUAGE OF ALGEBRA. 1960. sd, co, 16 min., jh, sh; International Film Bureau, \$165.

Beginning with a display of symbols like "stop" and "go" lights, highway markers, and other signs which give directions, this film goes on to identify symbols for things, names of people and places. The development of some symbols is reviewed, such as word and letter symbols. In spite of this introduction, the use of symbols in the remainder of the film is very confusing particularly with respect to constants, variables and numerals.

The language used in this film is mathematically poor. No mention is made of basic algebraic principles which results in statements like "2 is removed from the parenthesis." The reviewers feel that the film is not usable in algebra courses

LANGUAGE OF ALGEBRA. See Junior High Film Series.

LANGUAGE OF GRAPHS. 1948. sd, bw, 13 min., jh; Coronet Instructional Films, \$62.50, co \$150.

Practical uses of bar graphs, line graphs, circle graphs, and graphs of a function are illustrated. The value of a graph in telling a story with pictures is shown as well as the algebraic relations between x and y values on a straight line. Insight into some business principles is also offered.

This film will be of particular use to students studying some simple applications of mathematics.

THE LANGUAGE OF MATHEMATICS. 1950. sd, bw, 11 min., jh; Coronet Instructional Films, \$60, co \$120.

A fire drill is used as a starting point in a simple discussion of the use of precise information in mathematics to solve problems. The language of mathematics is shown to be a way of communicating ideas as in blueprints, graphs, and other forms.

The issues considered in the film suggest that a more appropriate title would have mentioned something about precise measurements.

Large Angles and Coordinate Axes. See Trigonometry Series.

LAW OF COSINES. See Trigonometry Series.

LAW of SINES. See Trigonometry Series.

LAW OF TANGENTS. See Trigonometry Series.

LINEAR EQUATIONS IN ONE UNKNOWN. See Advanced Algebra Series.

LOGARITHMIC OPERATIONS, I, II. See Engineering Problems.

LOGARITHMIC SYSTEMS. See Engineering Prob-

LOGARITHMS AND THE SLIDE RULE. 8 films, 1961. sd, bw; sh, jc, a; International Film Bureau, Inc., \$795.

This series of eight films might be usable in technical courses in which skills alone are desired. Certainly no one should use this series who wants to derive mathematical understanding of logarithms and the slide rule. The four films on slide rule are more satisfying than the four on logarithms probably because we view the slide rule as more of an instrument than a mathematical idea. The most serious criticism of the series is the disregard for standard notation and the reliance on rote learning of rules that are often very arbitrarily conceived.

Logarithms and the Slide Rule—Lesson I. 30 min., \$125; sh, jc, a.

The series opens with a general discussion of the need for speed in computation. The slide rule and digital computer are mentioned as important advances in the area of computation, and a typical slide rule is displayed as well as several other types including the spiral type with a 500-foot scale.

After a problem in regular multiplication, the rules of exponents are reviewed. The possibility of expressing numbers as powers of ten and multiplying by adding exponents is shown by example. It is not mentioned that 16.53 is only approximately equal to 101.21827. A definition of logarithm is given but the base b is not restricted as it needs to be for this discussion. For example, it should be pointed out that one is not a logical choice for b. Using the rules for exponents, the parallel rules for logarithms are presented. The traditional procedure for determining the characteristic by noting the powers of ten which bracket the given numeral is given but the rule "one less than the number of digits to the left of the decimal point" is entirely inadequate. A number "partly positive and partly negative" is invoked to explain the method for determining the characteristic of numbers less than one. Furthermore, it is not mentioned that these numbers are restricted to the non-negative numbers.

Logarithms and the Slide Rule—Lesson II. 30 min., \$125; sh, jc, a.

Following an extensive review of Lesson I, several examples are worked to practice using the rule for numbers greater than one. The reason given that numbers 68.0 and 680 have the same mantissa is that these numbers are "the same distance along the way" between 10 and 100, 100 and 1000, respectively.

The method of adding and subtracting 10 to a logarithm for convenience of operation is presented. Phrases such as "0.0068 is penned in between 0.01 and 0.001" and "a number farther

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in the hole" are used in the development which indicate the lack of precision in language and the general intuitive aspect of the series.

Logarithms and Slide Rule-Lesson III. 30 min., \$125; sh, jc, a.

An extensive review of the first two lessons is followed by a simplification of

$$\sqrt[8.5]{\frac{(6.278)(2.06)}{(92.563)(0.00806)}}$$

using a logarithmic scheme to ease computation. No justification is given in many cases with

complete dependence on rules.

An elementary method for devising entries in the logarithm table is presented. It is shown that logarithms are in arithmetic progression while the corresponding powers of ten are in geometric progression. The general development here is good although until this point the instructor has not mentioned that he deals with approximations.

Logarithms and the Slide Rule—Lesson IV. 30 min., \$125; sh, jc, a.

A lengthy review of the method for computing any power of ten could be considerably shortened by noting that, for example,  $10^{2.5} = \sqrt[2]{10^5}$ .

The extraction of the square root of 100,000 by means of the algorism is done in detail but does not seem to be of interest here and could be

deleted.

A logarithm table is examined carefully and the entries explained. An example is shown using the table but the definition of significant digits is inadequate. An example of the method of finding an anti-logarithm is carried out, but the rule for interpolation is simply given without any justification. In fact, interpolation is treated much too briefly and the student who knows little about the technique will find the film of no help.

Logarithms and the Slide Rule—Lesson V. 30 min., \$125; sh, jc, a.

The discussion of the slide rule begins in this film. The C and D scales are identified and the relative positions of the numerals on these scales are explained in terms of logarithms. In this explanation the lecturer says "the little 9 is less than the large 2" which leaves something to be desired. Several examples of multiplication are shown including some in which the problem of "going off the scale" appears.

Logarithms and the Slide Rule—Lesson VI. 30 min., \$125; sh, jc, a.

After a review of the previous film, the C and D scales are used in division and the "off the scale" problem is handled for division. The principle of proportion in the use of the slide rule is discussed in some detail and several problems involving proportions are solved. Examples

of problems in which both multiplication and division are involved are carried out with good technique. The rules for multiple settings on the slide rule are discussed.

Logarithms and the Slide Rule—Lesson VII. 30 min., \$125; sh, jc, a.

The CI scale is discussed and several problems involving this scale are solved. The use of the slide rule in reciprocals, multiplication, and division is demonstrated. The CI scale is emphasized because it shortens calculation time by reducing the number of settings required.

The A and B scales are explained and used

for squaring and extracting square roots.

Logarithms and the Slide Rule—Lesson VIII. 30 min., \$125; sh, jc, a.

The A scale, settings, square root, and result reading are reviewed. The rule for locating the decimal point is given but could be handled more easily through the use of scientific notation. The decimal point is "moved around to the left" indicating poor use of language. The K scale and cube roots are discussed but the method for locating the decimal point is not good.

LOGARITHMS—CHARACTERISTICS. See Engineering Problems.

McGraw-Hill Teacher Education Series. 5 films, 1959. sd, bw; jh, sh; tchrs. of el, jh, sh; guide; McGraw-Hill Text Films, \$600.

Sentences and Solution Sets. 21 min., \$140; ih, sh; tchrs. of el, jh, sh; guide.

The instructor points out that the concept of set plays a very basic role in our daily lives. He claims that the major advantages accruing from the use of sets in mathematics are clarification, simplification, and unification. By means of a somewhat artificial classroom scene, the idea of an open sentence is presented. The terms "set selector," "subset," "universal set," and "solution set" are illustrated. A variable is defined precisely and this definition is contrasted with the various vague descriptions given in many traditional algebra classes.

We recommend this film very highly for viewing by all teachers and prospective teachers of elementary and secondary mathematics.

Concept of a Function. 16 min., \$105; jh, sh, jc, sc; tchrs. of jh, sh; guide.

In this film the lecturer develops the notion of function from the set concept. Sentences in two variables are discussed including the graphs and solution sets of such sentences. This background serves as preparation for the definition of function which is defined as a set of ordered pairs such that no first element can appear with different second elements. Domain, range, and rule of a function are defined with emphasis on the point that a function is not a formula but rather a set of ordered pairs. The expression



F(x) is defined as the value of the function at x or the value of y. We feel it would be more consistent with the definition of function to define it as the second element of the ordered pair whose first element is x.

The lecturer succeeds in presenting sound mathematics and uses his time efficiently. Although the films are designed for teacher training we feel this film could be used with high school students who have some background in contemporary mathematics.

Irrational Numbers. 23 min., \$150; jh, sh, jc, sc; tchrs. of jh, sh; guide.

"An irrational number is the square root of a number that we can't take the square root of." The lecturer submits that a statement such as this would more often than not summarize an algebra student's knowledge of irrational numbers. He then demonstrates, with the help of classroom scenes, that the road to far greater understanding is not a rocky one and is certainly worth traveling.

The film emphasizes the importance of the study of the decimal expansions of numbers. A rational number is defined as a number which is the quotient of two integers. Also discussed are properties of order and denseness in the set of real numbers.

Except for a puzzling statement about the period of a repeating decimal, namely: "We say the decimal has period 2 because it starts to repeat after the second place," this film is well done. It should prove quite valuable if put to use in preservice and in-service teacher training.

Number Fields. 17 min., \$115; sh, jc, sc; tchrs. of jh, sh; guide.

The film opens with a dialogue between a student and his teacher concerning rationalizing the denominator of  $(12-7\sqrt{3})/(3-2\sqrt{3})$ . The scene then shifts to the regular lecturer who indicates that the teacher would have been better prepared to give answers to such questions as "Why do we rationalize the denominator? Will we always get an answer of the form  $a+b\sqrt{3}$ ?" if she had been well versed in the concept of number fields.

Then follows a quick review of the concept of set and properties of an operation. A definition of closure is carefully given. A field is defined and the laws concerning its operations are listed.

The set of integers is used as an example of a set which is not a field. The rational numbers, real numbers, and complex numbers are shown to be fields before returning to the numbers whose numerals are of the form  $a+b\sqrt{3}$  and this set is shown to be a field also.

The presentation is well done but we feel that number fields is too large a topic for a film of this length. We also feel that better motivation could have been provided for the topic although this does not detract from its effectiveness. The film can be used for in-service education or teacher training if the instructor is careful to give further examples including finite fields.

Patterns in Mathematics. 14 min., \$90; jh, sh; tchrs. of jh, sh; guide.

The opening scene of the film shows the ininstructor in a discussion of the essential nature
of mathematical problems. He notes that while
the mathematician looks for patterns, the high
school student often looks at mathematics as a
bag of tricks. High school mathematics should
be a study of patterns, says the instructor, since
this is the essence of the "new mathematics."
Some of the patterns discussed are the distributive law, commutative law for multiplication,
graphs of straight lines, and the solving of simultaneous linear equations. The point is made
that patterns develop ability to generalize,
understand relationships, improve insight,
power, and readiness for further study.

This would be an excellent film for teachereducation classes or seminars. The reviewers are especially pleased by the accuracy of the mathematics in this film and series.

Mantissas. See Engineering Problems.

THE MEANING OF PI. 1949. sd, bw, 12 min., jh, sh; Coronet Instructional Films, \$60, co. \$120.

The terminology for circles is introduced and illustrated with several objects. It is shown that it takes a "little more" than three diameters to give the circumference of a circle and it is seen that as diameter increases, the circumference increases. Further comparison shows that the ratio of circumference to diameter is slightly more than 3.14. A brief history of several notations for expressing  $\pi$  is given. It is emphasized that the value  $3\frac{1}{7}$  is chosen for convenience only.

More Solutions of Linear Equations. See Intermediate Algebra Series.

MULTIPLICATION AND DIVISION. See Engineering Computation Skills: The Slide Rule.

MULTIPLICATION OF RATIONAL NUMBERS. See Intermedia Algebra Series.

Mysterious X. See Adventures in Number and Space.

NATURE OF LOGARITHMS. See Advanced Algebra Series.

NEW NUMBERS. See Understanding Numbers.

Number Fields. See McGraw-Hill Teacher Education Series.

THE NUMBER SYSTEM AND ITS STRUCTURE. 1961. sd, co, bw, 11 min.; jh, sh; Coronet Instructional Films, bw, \$60, co. \$110, guide.

After a brief history of number which includes the notion of place holder, concepts concerning the number system are discussed. The

property of closure, the commutative and associative laws, and the distributive law are in-

cluded in this discussion.

For a review of our number system's structure and fundamental principles, this film should be excellent. However, if used as an introduction, too much will be covered to allow good understanding. There are some errors which will need correcting by the teacher.

NUMERATION SYSTEMS. See Junior High Film Series.

OBLIQUE TRIANGLES. See Engineering Computation Skills: The Slide Rule.

PATTERNS IN MATHEMATICS. See McGraw-Hill Teacher Education Series.

PERMUTATIONS AND COMBINATIONS. See Advanced Algebra Series.

PRACTICAL USE OF LOGARITHMS. See Trigonometry Series.

Progressions, Sequences, and Series. See Intermediate Algebra Series.

PROPORTIONS AT WORK. 1961. ad, co, bw, 12 min., jh, sh; International Film Bureau, Inc. \$120.

A biologist at work using proportions serves as motivation for this film. A vague definition of ratio is given and some properties of proportions are dealt with briefly. The rule that the product of means equals product of extremes is derived but done too quickly for most students to follow easily. Units of linear and area measure are included but their use in proportions follows the pattern used in many physics courses.

The film does not seem to cover too much material and, if used, could promote considerable classroom discussion in general mathema-

tics or algebra classes.

PYTHAGOREAN THEOREM: THE COSINE FOR-MULA. 1960. sd, bw, 5½ min., sh; Coronet Instructional Films, \$30.

This film uses animation to illustrate the derivation of the law of cosines. The Pythagorean theorem is then derived as a special case of this law by algebraic methods and by geometric animation.

The pace of the film may be too fast for average students and the development may be too rigorous for most students to follow if used

as an introduction.

Pythagorean Theorem: Proof by Area. 1960. sd, bw, 5½ min.; jh, sh; Coronet Instructional Films, \$30.

The Pythagorean theorem is stated and a special case of an isosceles right triangle is considered. The equality of the areas of parallelograms with equal bases and altitudes is con-

sidered and the notion used to justify the Pythagorean theorem for any right triangle. Examples are illustrated.

The animated demonstrations given are good but of course, do not constitute proofs. The pace of the film is probably too fast for the average eighth grader who sees these ideas for the first time but might be adequate for reviewing.

QUADRATIC EQUATIONS. See Intermediate Algebra Series.

QUICKER THAN YOU THINK. See Adventures in Number and Space.

RADICALS AND THE REAL NUMBER SYSTEM. See Intermediate Algebra Series.

RAISING NUMBERS TO POWERS. See Engineering Computation Skills: The Slide Rule.

RATIO PROBLEMS. See Engineering Computation Skills: The Slide Rule.

THE RATIONAL NUMBERS. See Junior High Film Series.

RIGHT TRIANGLE APPLICATIONS. See Engineering Computation Skills: The Slide Rule.

RIGHT TRIANGLES. See Engineering Computation Skills: The Slide Rule.

RIGHT TRIANGLES AND TRIGONOMETRIC RATIO. See Trigonometry Series.

ROOTS AND EXPONENTIAL EQUATIONS. See Engineering Computation Skills: The Slide Rule.

ROOTS OF HIGHER ORDER. See Intermediate Algebra Series.

SENTENCES AND SOLUTION SETS. See McGraw-Hill Teacher Education Series.

SHORT CUTS. See Understanding Numbers.

SIGN LANGUAGE. See Adventures in Number and Space.

Similar Triangles in Use. 1961. sd, co., 11 min., jh, sh; International Film Bureau, Inc., \$120.

Two examples of the use of similar triangles and their proportional sides are given to illustrate practical applications. Trigonometry is coupled with similar triangles and some of the special tools of occupations using these ideas are shown

It is probably the case that most teachers could stage situations that would be at least as effective as those in the film. The emphasis on trigonometry as a tool of the occupations illustrated is probably misplaced in the light of present use of trigonometric functions.

SIMPLIFTING COMPLEX FRACTIONS. See Advanced Algebra Series.

SLIDE RULE. See Special Lessons in Physics.

Solids in the World Around Us. See Discovering Solids.

SOLUTION OF EQUATIONS BEYOND THE SECOND DEGREE. See advanced Algebra Series.

Solving Equations in Fractional Form. See Intermediate Algebra Series.

SOLVING PROBLEMS WITH THE QUADRATIC FORMULA. See Advanced Algebra Series.

Solving Simultaneous Linear Equations. See Intermediate Algebra Series.

Special Lessons in Physics. 3 films, 1957. sd, bw, sh; tchrs. of sh; Encyclopedia Britannica Films, Inc., bw, \$495 co \$990; guide.

Elements of Trigonometry. 30 min., sh; tchrs. of sh; bw \$165, co. \$330.

Ideas on significant figures are discussed at the outset, but unless the viewer is already familiar with these notions, we feel he will have some difficulty in following the discussion. A large demonstration slide rule is effectively used to examine the parts and scales of the slide rule. Multiplication and division are carefully discussed as well as the computation of squares and square roots.

The closeups of the slide rule are better than would be possible in the usual classroom demonstration. No attempt is given to show the rationale behind the processes. We feel that the film might be useful as an introduction to the slide rule but that the pace of presentation is too

fast.

Algebra and Powers of Ten. 30 min., sh, tchrs. of sh; bw \$165, co. \$330.

Equations as pictures of experiments are considered. A balance is used to demonstrate some transformations of special equations. A term of the type  $\frac{1}{4} + \frac{1}{1}$  is simplified. Some common errors that occur in the treatment of literal equations are pointed out. A model is shown in an attempt to illustrate the square of a binomial. A definition of exponent and several examples with base ten are discussed. Scientific notation is discussed as a practical way of writing very great or very little numbers.

The use of a balance in transforming equations is not very good. The treatment of fractional equations involves much symbol juggling. The model used to illustrate the square of a binomial needs dark and light shading of its parts since it is difficult to see. Scientific notation is very hastily introduced and, although examples are cited, no definition of negative exponents is given. The definition given for exponent is very poor. We do not recommend that this film be used in secondary schools.

Slide Rule. 30 min., sh; tchrs. of sh; bw \$165, co. \$330.

An angle is defined and a specific angle is constructed. Using the right triangle, definitions of sine, cosine and tangent are given. If a and b are the acute angles of a right triangle then it is seen that  $a+b=90^{\circ}$  and  $\sin a=\cos b$ . An experiment is performed to examine the ratio of sides to fixed hypotenuse of 50 centimeters. Projections of line segments on a given line are discussed.

The visual aids used in the film are very good. The material presented will be incomplete for those who have never studied trigonometry and is, in general, hastily presented. After the experiment described above, no mention is made of the purpose of a table of such ratios. The film would do a fair job of reviewing sine, cosine, and tangent.

SPECIAL PRODUCTS AND FACTORING. See Intermediate Algebra Series.

SQUARES, CUBES, AND ROOTS. See Engineering Computation Skills: The Slide Rule.

STANDARD TECHNIQUES OF FACTORING. See Advanced Algebra Series.

STRETCHING IMAGINATION. See Adventures in Number and Space.

SURFACE AREAS OF SOLIDS I AND II. See Discovering Solids.

SYMBOLS IN ALGEBRA. 1961. sd, bw; jh, sh: Coronet Instructional Films, bw \$60, co. \$120 11 min.; guide.

An introduction shows that the students have been using formulas in arithmetic and that the basic purpose of algebra is the establishing of general rules such as these formulas. The film shows how letter symbols are used in much the same way as numerals in arithmetic and concludes with an example showing the use of a letter symbol as an unknown in an equation.

The best use of this film would be to introduce a unit on algebra in Grade 8 since it does a good job of relating algebra to arithmetic. The solving of equations is handled only briefly.

TABLES OF TRIGONOMETRIC RATIOS. See Trigonometry Series.

THEORY OF EQUATIONS AND SYNTHETIC DIVISION. See Advanced Algebra Series.

TIME. 1959. sd, co, bw, 15 min.; el, jh, sh, jc, a; tchrs. of el, jh, sh; Indiana University, bw \$75, co. \$150.

The use of time in daily living is the theme of this film. The sun is shown to be one of man's oldest time pieces. A detailed treatment is given to the development of the time zones with an

animated sequence of a rocket circling the earth being used to illustrate the necessity for the International Date Line. Daylight Saving Time is explained. A discussion follows on how astronomers take photographs of stars' paths to determine time and the film concludes with a summary of the ideas discussed.

The animation and models are excellent. The inclusion of an examination of pendulums is well done and the familiar examples given are good. The use of a rocket to show the need for the International Date Line makes this idea easy to understand. This is one of the best films

this committee has seen.

TRIGONOMETRIC APPLICATIONS. See Engineering Problems.

TRIGONOMETRIC RATIOS AS PERIODIC FUNC-TIONS. See Trigonometry Series.

TRIGONOMETRIC SCALES. See Engineering Computation Skills: The Slide Rule.

TRIGONOMETRY AND SHADOWS. See Trigonometry Series.

TRIGONOMETRY OF LARGE ANGLES. See Trigonometry Series.

TRIGONOMETRY MEASURES THE EARTH. See Trigonometry Series.

TRIGONOMETRY SERIES. 21 films, 1960. sd, bw, sh, jc; Modern Learning Aids, \$3,150.

This series may be useful in technical, vocational, or service courses where the theory underlying trigonometric skills is unimportant. There are a few exceptions to this statement as will be noted in the individual film reviews, but it is generally the case that skill in techniques receives greatest emphasis. Therefore, the series is not to be recommended for contemporary

curriculum programs.

Pedagogically, we feel that too much information is given to be memorized. Certainly some facts need to be memorized but the quantity required here is excessive. There are inconsistencies in notation, for example, 'N.D.' is used for the words 'not defined' in an early film and the symbol for infinity is used later. The emphasis on what not to do is questionable and yet stress is often placed on procedures that are not to be done. Repetition is also useful in teaching when used sparingly but often items are repeated to the limit of boredom. It seems to this committee that the most important constructions should be made in the presence of the students and yet in these films it is often the case that these constructions are already on the chalk-board at the beginning of the film. This leaves the viewer with no idea as to the development of the notion. This committee objects to the colloquial language used in the series. Such terms as "shuffling fractions," "divided out," "penned in," "knock out," "cleaning up," "opening parenthesis," "this number is really real," "the sine of zero is nothing at all," "a segment is capable of," "the square roots will lift off," and many more, are not acceptable in a series designed for wide distribution.

Definitions are, in general, poorly stated, if indeed given at all. The logical structure of trigonometry is not well presented in spite of the stated intent of making trigonometry an extension of geometry. No clear distinction is made between convention and definition. As seen above, the language used is imprecise and the mathematics suffers as a consequence.

Trigonometry and Shadows. 26 min., \$150; sh, jc, a.

A review of various applications of trigonometry—surveying, construction, navigation, warfare, and cartography—is given briefly around the early history of trigonometry. The significance of the ratio of the lengths of the sides of a triangle is stressed, and the sine of an

angle is defined.

The historical comments at the beginning of the film are good although most teachers would be able to provide similar comments of their own. The drawing illustrating Thales' solution to measure the inaccessible height of a pyramid seems to be too intricate. The notion of a standard triangle comes very fast and it will be difficult for most students to follow. The ideas of sine as a ratio is not the notion which is currently being used. This is also true of the other functions which are discussed here.

Right Triangles and Trigonometric Ratio. 29 min., \$150; sh, jc, a.

Thales' method for finding the height of a pyramid using the ratio of corresponding sides of similar triangles is generalized in the definitions of the sine, cosine and tangent of an angle as ratios of the sides of a triangle having one right angle. The values of these functions for angles of 30, 45, and 60 degrees are derived.

This is an extension of the traditional treatment found in the first film. The statement that the square root of 3 is 1.732 implies that the square root of 3 is a rational number. Other errors are noted such as a line segment being called a ratio. Defining "function" as "depends on" is extremely confusing. This presentation of trigonometry will hardly be in line with any of the contemporary programs.

Using Sines, Cosines and Tangents. 29 min., \$150; sh, jc, a.

After a brief review of the definitions and values of the sine, cosine, and tangent functions for 30, 45, and 60 degree angles, a series of examples is demonstrated. Emphasis is placed on proper choice of function to make the solution easiest. An arc of a unit circle is defined and used to expand the relationships among the simple trigonometric functions and to show that these functions are linear.

The instructor wisely guides the observer to consider the plausibility of answers that are obtained and uses some good illustrations. The unit circle is used in the classic manner to define the functions rather than to use the unit circle coordinates and a winding function. The nonlinearity of the tangent function is not made clear. The accuracy of the statements about the history of the word "sine" is doubtful.

Trigonometry Measures the Earth. 28 min., \$150; sh, jc, a.

The film illustrates the power of trigonometry in the solution of complex and difficult problems. It shows in some detail how Eratosthenes measured the circumference and diameter of the earth. A method of finding the distance to the moon is also described.

The reaction of the reviewers to this film is favorable and they recommend its use in any of the different types of trigonometry courses—contemporary or traditional. It provides excellent historical interpretation of the applications of trigonometry. It is suggested, however, that the teacher using this film carefully preview it and make reference to the fact that the lecturer does not refer to the diagrams as triangles. The use of props is definitely primitive.

Cosecant, Secant and Cotangent. 27 min., \$150; sh, jc, a.

The cosecant, secant, and cotangent functions are defined and shown to be the reciprocals of the sine, cosine, and tangent functions. The representation of these functions as line lengths associated with the unit circle is described in some detail and the origin of the names of the various functions is given. Values of these functions are developed for angles of 0, 15, 30, 45, 60, 75, and 90 degrees. The complete range of trigonometric tables is hinted at but not yet developed.

Inconsistency in language is persistent. The terms "trigonometric functions," "trigonometric relationships," "trigonometric ratios," and even "trigonometric segments" are used synonymously. The observer may derive the impression, from the phraseology of the instructor, that sine is an increasing function and cosine is a decreasing function.

Eight Fundamental Trigonometric Identities. 28 min., \$150; sh, jc, a.

This film was not reviewed. However, the publisher's description is given here.

The Pythagorean relation for right triangles is used to develop three trigonometric identities involving the squares of the simple trigonometric functions. The two ratio relationships for tangent and cotangent in terms of sine and cosine, and then the three reciprocal relationships defined in the previous film complete the eight fundamental trigonometric identities. Several examples of the use of these identities for simplifying complex trigonometric relationships are worked out in step-by-step detail.

Working With Trigonometric Identities. 29 min., \$150; sh, jc, a.

A brief review of the eight fundamental identities serves as a basis for analysis of more complicated identities. Techniques such as working on only one side of the identity, watching the form of the terms, and checking results are shown in detail. Geometric illustrations of the identities are developed on the unit circle.

The instructor does not make it clear that identities are theorems and fails to write these proofs in such a way that the form of the proof is clear. The fact that the domains of the eight fundamental identities have not been restricted allows for some errors in conclusions. There are also occasional errors on the chalkboard both in mathematics and in pedagogical efficiency. The terminology is often colloquial and idiomatic and, in too many cases, the language is not precise.

Tables of Trigonometric Ratios. 29 min., \$150; sh, jc, a.

Conventional tables of trigonometric functions are described and examples of their use illustrated. This film could be used in some courses to supplement a lecture on the use of trigonometric tables.

The use of materials in this film is good. On the other hand, the pedagogy is questionable, particularly when stress is placed on things that should not be done. Moreover, great emphasis is placed on memorization of large amounts of material.

Interpolation in Trigonometric Tables. 28 min., \$150; sh, jc, a.

After a review of the form and organization of trigonometric tables, the technique of reading such tables is described with care. Examples are worked out in detail, showing how to carry out interpolation and inverse interpolation procedures. The importance of careful organization of one's work, and the errors to be watched for and avoided, are considered.

This film is a good summary of the use of trigonometric tables. The methodology in the film indicates that the development is reflecting a cookbook version of trigonometry. This is particularly shown by the rigorous patterns that are required for interpolation.

Introduction io Logarithms. 28 min., \$150; sh, jc, a.

Logarithms are introduced as a useful mathematical technique for carrying out the computational manipulations required in solving complex trigonometry problems. A logarithm is first defined as an exponent in the general sense that if  $N=B^p$ , then  $\log_B N=P$ , where P is the logarithm of N to the base B. Examples of the use of logarithms are then worked out, and in the process the definitions of mantissa and characteristic are developed.

The first rule for determining the characteristic involves "counting the number of digits to the left of the decimal point, and subtracting one."

Practical Use of Logarithms. 30 min., \$150; sh, jc, a.

Various applications of the use of logarithms are presented including problems of multiplication, division, raising to powers, and extracting roots. All discussion is restricted to base ten and the emphasis is on the service aspect of trigonometry rather than as an important branch of mathematics.

Using Logarithm Tables. 29 min., \$150; sh, ic. a.

Finding the mantissa of a logarithm from a three-place table and then the reverse process of finding a number given its logarithm, is shown. A brief discussion of significant digits is applied to logarithms and antilogs. Examples are worked out showing how to interpolate in log tables, and how to do inverse interpolation. The log of a trigonometric value is found in a table, and the existence of log trig tables is presented briefly. Finally, the nature of the scales and the use of a slide rule are discussed.

The major part of the film is devoted to the continuation of the study of logarithms that was begun earlier. The section on the nature and use of the slide rule will provide the user with no help in teaching the slide rule.

Large Angles and Coordinate Axes. 30 min., \$150; sh, jc, a.

Many applications of geometry and trigonometry involve angles greater than 90°. This film defines and describes these large angles and their trigonometric values. Positive and negative angles, the four quadrants of a plane, and the coordinates of a point in any of the four quadrants are discussed. The values of the sine, cosine, and tangent (with proper signs) are defined for all quadrants. Methods of reducing the large angles to equivalent smaller ones for the use of trigonometry tables are explained.

Trigonometry of Large Angles. 30 min., \$150; sh, jc, a.

After a review of the nature of large angles, it is shown that all the definitions of the trigonometric functions in terms of R, x, and y apply to any large angle if proper care is taken to identify the sign of the function. Thus, trigonometric functions are signed quantities. All eight of the simple trigonometric identities are shown to work in all four quadrants. Finally, a number of examples are worked out in detail showing the reduction of functions of large angles to small, and of functions of negative angles to equivalent functions of positive angles.

Again, there is a tendency to ask the student to memorize too much extraneous material, such as the "CAST" rule for remembering the positive functions in each of the four quadrants.

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Law of Sines. 30 min., \$150; sh, jc, a.

The equation for the Law of Sines is derived and then used to solve a problem in which two angles and the included side are known. The solution provides directly the values for all three sides and all three angles. Finally, an ambiguous problem is described in which are given two sides and the angle opposite one of them. Either of two solutions is possible. A single solution would occur if one unknown angle is a right angle.

The reviewers feel that this film could possibly be used separately from the sequence. However, there is a continuing use of sloppy language. The derivation of the Law of Sines may be a bit sophisticated. It is geometric in nature, however, and in this respect, ties in well with background materials in the other films of the series.

Law of Cosines. 30 min., \$150; sh, jc, a.

A derivation of the Law of Cosines is shown and its application to a problem is carefully worked out. The problem of three given sides is worked out. The tedious calculation is pointed out and the advantages of logarithms in such situations are made obvious. However, the Law of Cosines is not written in a form suitable for the use of logarithms.

This film might be used as a supplement to the "in-class" teaching of the Law of Cosines. The film is not as good as the one on the Law of Sines. Extreme monotony arises in this film because of the large amount of repetition. The film would be more useful if it had been more carefully edited.

Law of Tangents. 28 min., \$150; sh, jc, a.

The Law of Tangents is presented and used in a practice problem. The derivation of the Law of Tangents is quickly done with the help of previously prepared figures and equations.

Law of Tangents could be used as a supplementary film to a lecture on the Law of Tangents or as a filler on that particular topic in case the teacher chooses this method. It is a good film for enrichment for able students as it presents an unusual geometric proof of the Law of Tangents.

Trigonometric Ratios as Periodic Functions. 28 min., \$150; sh, jc, a.

Conic sections, periodic and harmonic motion, and the general form of the sine curve are discussed. The general presentations seem to be satisfactory although since this is the first consideration of the sine function as a periodic function, the development may be difficult for the average student to follow. This is one of the unfortunate consequences of this kind of development. Several definitions are not clearly given and such terms as "period of a function" and "amplitude" are only incidentally mentioned. The graph of  $y = A \sin Bx$  is not carefully

explained. The film may be useful in some situations but the pace will be too fast for most students.

Graphs of Periodic Functions. 29 min., \$150; sh, jc, a.

Further discussion on  $y = A \sin Bx$  is presented as well as radian measure. Plotting periodic functions in terms of radians is described, and the sine curve is contrasted with the tangent curve.

The use of this film is not recommended in any class because of the extremely poor terminology that is used in it. To cite an example, the amplitude of a sine curve is defined as "the thickness and thinness of the up and down." The relation  $s = x \cdot r$  (where s represents the arc length, x the number of radians in the central angle, and r the radius) is given without any appeal to intuition or proof. The student is asked to memorize that " $\pi$  is equal to 180° and later the statement is given that  $\pi$  radians is equal to 180°.

Addition Formulas and DeMoivre's Theorem. 28 min., \$150; sh, jc, a.

This film is a rapid survey of several applications of trigonometry to algebraic equations. Complex roots of a cubic equation are derived from DeMoivre's Theorem which is itself derived from addition formulas for trigonometric functions.

The film covers entirely too much material to be conveniently understood by most students. DeMoivre's Theorem is not well established and the formula for the sine of the sum of two angles is developed for first quadrant angles only.

Double and Half Angle Formulas. 28 min., \$150; sh, jc, a.

The addition formula developed in the previous film is used to derive the double-angle formulas for sine, cosine, and tangent. The use of these formulas in simplifying identities is worked out in a problem. The half-angle formulas for sine, cosine, and tangent are derived and used to simplify another identity.

This film presents nothing that could not be presented by a regular classroom teacher. The developments are standard for the double and half-angle formulas. In the lecturer's summary, he mentions that he has tried to illustrate the logical and psychological unity of trigonometry. The logical unity was certainly not apparent; neither was the psychological!

UNDERSTANDING NUMBERS. 7 films. sd, bw; jh, sh, jc, sc, a: tchrs. of el, jh, sh; University of Michigan TV, \$700.

The Earliest Numbers. 30 min., \$100; jh, sh, jc, sc, a; tchrs. of el, jh, sh.

Some early numeration systems are discussed with emphasis on the Egyptian and

Babylonian systems. A good demonstration of the use of a counting board in computation is given. The relationship between number and language is considered.

The historical development is well done, although at one point the Egyptian and Babylonian systems were interchanged and at a later point in the film Arabic symbols were used in place of Egyptian symbols. No differentiation was made between symbols used to compute and symbols used to record in the systems discussed. This committee feels that the film definitely covers too much historical ground and that the summary, which is given orally by the instructor, fails to tie up the ideas given. Overall, the film is well done in spite of these weaknesses and it would certainly be worthwhile as a film on the history of numeration systems.

Base and Place. 30 min., \$100; sh, jc, sc, a: tchrs. of el., jh, sh.

A presentation of the binary system and a demonstration of the use of the system in the digital computer are given in this film. The most outstanding feature of the film is the fine treatment of the binary system and its relationship to the development of digital computers. The lecturer fails to mention the relationship between the base and the number of distinct digits.

Big Numbers. 30 min., \$100; jh, sh, jc, sc, a; tchrs. of el, jh, sh.

The film illustrates and demonstrates the use of scientific notation. A rather extensive discussion of perfect numbers is also included.

The title of the film is slightly misleading since the discussion also centers on lesser as well as greater numbers. As a teaching instrument, the film will probably not serve too well, although it might be used in a mathematics club or for any occasion of general interest. The statement that only fifteen perfect numbers are known is no longer correct and will clearly date the film.

Fundamental Operations. 30 min., \$100; jh, sh, jc, sc, a; tchrs. of el, jh, sh.

The fundamental operations of addition, multiplication, subtraction, and division are the main issues here as well as the postulates associated with each of these operations. Subtraction is shown to be the inverse of the operation addition and similarly for multiplication and division. Addition and multiplication tables are given for modulo 5 and used to carry out a few exercises. Discussion is briefly extended to the rational numeration systems.

We recommend the film as a review of a section on modular arithmetic or as a preview of such a section. The emphasis on the patterns and fundamental operations in mathematics and the use of modular arithmetic to demonstrate these is good. The analogy of a "commutor" and the commutative law did not seem

satisfactory to this committee. The tables for addition and multiplication in modulo 5 are introduced without sufficient development leaving the student to wonder where they came from.

Short Cuts. 30 min., \$100; sh, jc, sc, a; tchrs. of jh, sh.

This film explains and illustrates some mathematical short cuts as a means of simplifying computation. The grating or gelosia method of multiplication, the principles of logarithms,

and the slide rule are considered.

We suggest that this film be used for review rather than as an introduction to the topics given here. Use is made of the notion that logarithms are exponents prior to the statement of this relationship. The instructor mentions that to multiply numbers it is necessary to add exponents, but neglects to restrict this operation to numbers with a common base. No summary is given and the over-all organization of the material is poor.

Fractions. 30 min., \$100; jh, sh, a; tchrs. of el, jh, sh.

The fable concerning the distribution of nineteen head of cattle by the portions \( \frac{1}{2}, \frac{1}{2}, \) and \( \frac{1}{2} \) is used to introduce fractions. A common fraction is defined and its five different meanings are analyzed with visual aids. The history of fractions is reviewed from the viewpoint of number theory. A chart is used to illustrate several types of fractions such as decimals, rational fractions, and duodecimal fractions. Some efforts are made to elucidate the meaning of the fractions and the relations between them. Attention is also given to repeating decimals.

The exploration of the five implied meanings of a fraction is adequate and stimulating with the use of visual aids being very helpful. The brief review of the history of fractions is satisfactory. The film seems appropriate for a course in mathematical appreciation for a group with widely different preparation in mathematics. The level of material is uneven and the range of difficulty is wide, although the presentation of the topics is not rigorous. Little attention is given to operations with, and applications of, fractions. The lecturer makes occasional slips in writing and in talking which indicate a need for more editing.

New Numbers. 30 min., \$100; sh, jc, sc, a; tchrs. of sh.

This film introduces the student to some of the new numeration systems. Rational, irrational, complex, and transfinite numbers are discussed in some detail. A good demonstration of the meaning of a one-to-one correspondence is given. The film provides the student with an

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excellent opportunity to consider numbers other than the familiar real numbers. A great deal of emphasis is placed on the proper naming of numbers.

The lecturer occasionally uses the word "number" when he should say "numeral." Many of the mathematical terms used, "cardinal" for example, are casually mentioned without adequate development or further use. The oral presentation is too rapid and the chalkboard work is not up to the par established in the other films of the series.

Using Fractional and Rational Exponents. See Intermediate Algebra Series.

Using Logarithms in Problems. See Advanced Algebra Series.

Using Logarithm Tables. See Trigonometry Series.

Using Logarithms to Solve Equations. See Intermediate Algebra Series.

Using Sines, Cosines and Tangents. See Trigonometry Series.

VARIATION: A LESSON IN READING. See Intermediate Algebra Series.

VOLUME AND ITS MEASUREMENT. 1960. sd, bw, 11 min.; el, jh; guide, Coronet Instructional Films.

Beginning with definitions of volume, formulas for volumes of rectangular solids, prisms, and pyramids are developed using plastic models. The need for a unit of measurement is stressed. Although the development of the volume formulas for rectangular solids and triangular prisms was incomplete, the film might be useful in junior high school classes on intuitive geometry.

Volumes of Cubes, Prisms, and Cylinders. See Discovering Solids.

VOLUMES OF PYRAMIDS, CONES, AND CYLINDERS. See Discovering Solids.

What's the Angle? See Adventures in Number and Space.

THE WHOLE NUMBERS. See Junior High Film Series.

WORKING WITH POSITIVE AND NEGATIVE Ex-PONENTS. See Intermediate Algebra Series.

WORKING WITH TRIGONOMETRIC IDENTITIES. See Trigonometry Series.